Electrical Engineering 1
12026105

Chapter 3

Methods of Analysis

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Understand Kirchhoff's current law.
2. Understand Kirchhoff's voltage law.
3. Develop an understanding of how to use Kirchhoff's current law to write nodal equations and then to solve for unknown node voltages.
4. Develop an understanding of how to use Kirchhoff's voltage law to write mesh equations and then to solve for unknown loop currents.
5. Explain how to use *PSpice* to solve for unknown node voltages and currents.

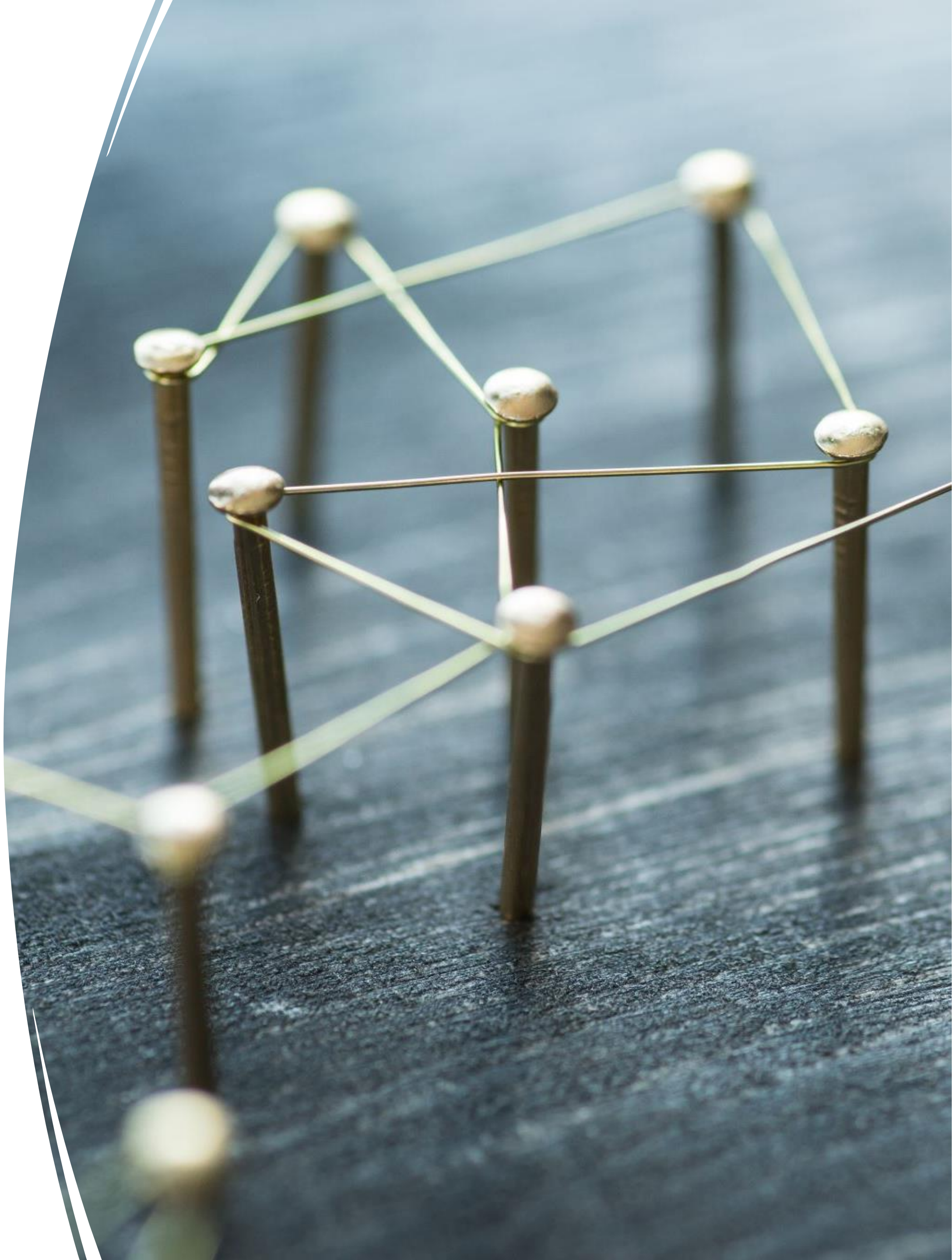
วัตถุประสงค์การเรียนรู้

โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:

1. เข้าใจกฎกระแสไฟฟ้าของ Kirchhoff
2. เข้าใจกฎแรงดันไฟฟ้าของ Kirchhoff
3. พัฒนาความเข้าใจเกี่ยวกับวิธีการใช้กฎกระแสไฟฟ้าของ Kirchhoff เพื่อเขียนสมการโหนด (node)แล้วหาแรงดันไฟฟ้าที่โหนดไม่ทราบค่าได้
4. พัฒนาความเข้าใจเกี่ยวกับวิธีการใช้กฎแรงดันไฟฟ้าของ Kirchhoffเพื่อเขียนสมการเมช (mesh)แล้วหากระแสไฟฟ้าที่ลูปที่ไม่ทราบค่าได้
5. อธิบายวิธีใช้ LTSpice เพื่อแก้ปัญหาสำหรับแรงดันไฟฟ้าของโหนดและกระแสไฟฟ้าที่ไม่ทราบค่าได้

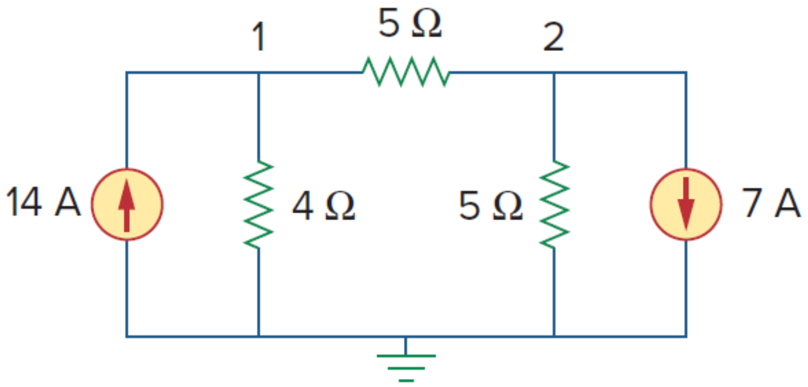
Methods of Analysis - Chapter 3

- Introduction
- Nodal analysis.
- Nodal analysis with voltage sources.
- Mesh analysis.
- Mesh analysis with current sources.
- Nodal and mesh analysis by inspection.
- Nodal versus mesh analysis.



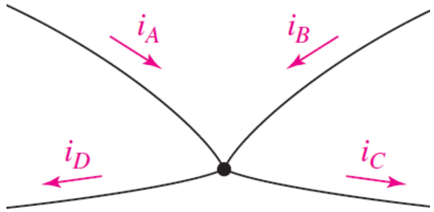
3.1 Introduction (1)

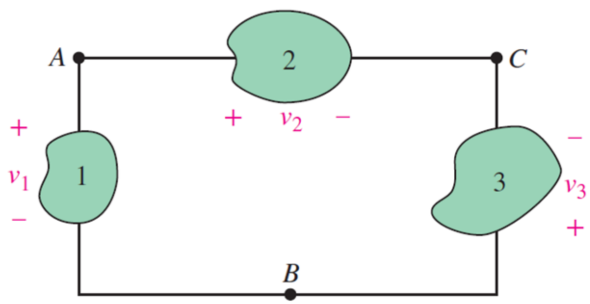
- If you are given the following circuit, how can we determine
  - Voltage across each resistor.
  - Current through each resistor.
  - Power generated by each current source, etc.

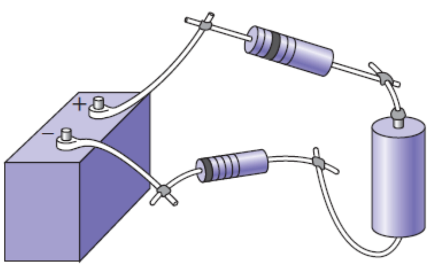


What are the things which we need to know to determine the answers?

3.1 Introduction (2)







KIRCHHOFF'S CURRENT LAWS (KCL)

KIRCHHOFF'S VOLTAGE LAWS (KVL)

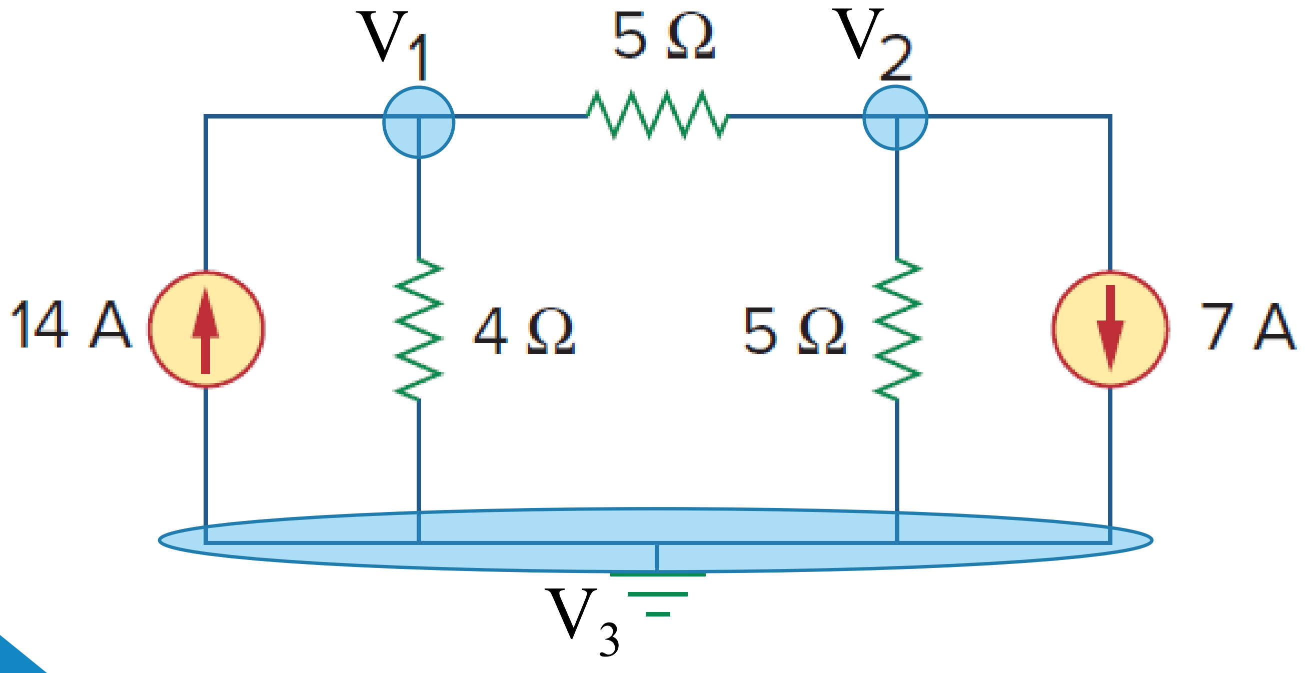
OHM'S LAW

How should we apply these laws to determine the answers?

จะใช้กฎเหล่านี้หาคำตอบของวงจรไฟฟ้าได้อย่าง?

3.2 Nodal Analysis (1)

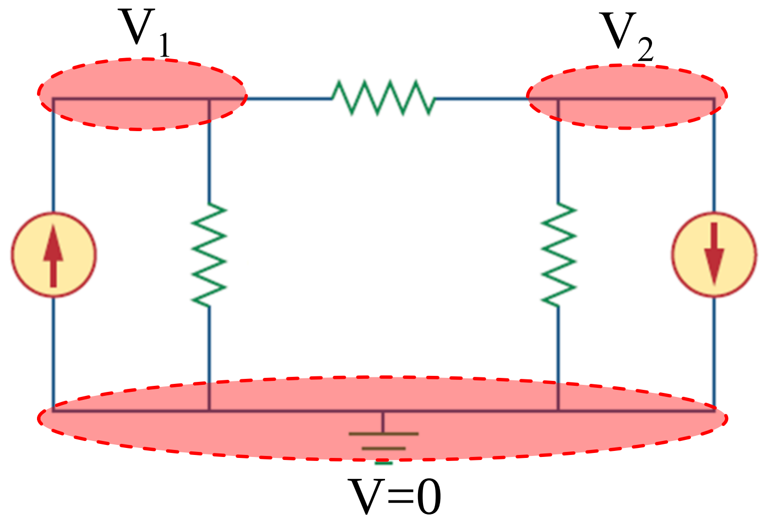
Ex. Analyzing circuits using *node voltages* as circuit variables.



3.2 Nodal Analysis (2)

Steps to Determine Node Voltages:

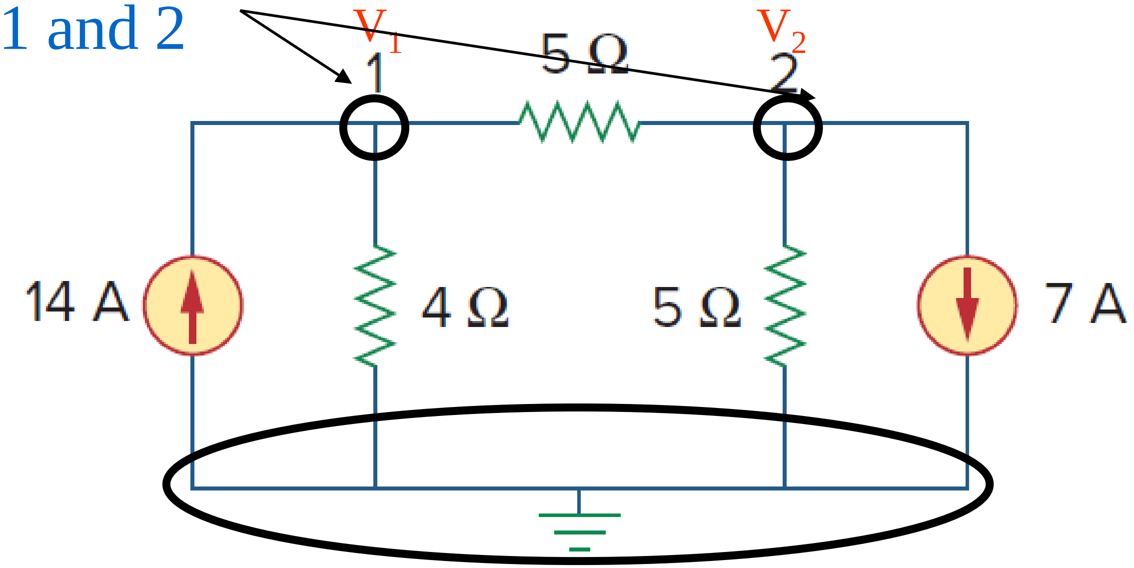
1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.



3.2 Nodal Analysis (3)

<u>Ex.1</u> Circuit independent current source only

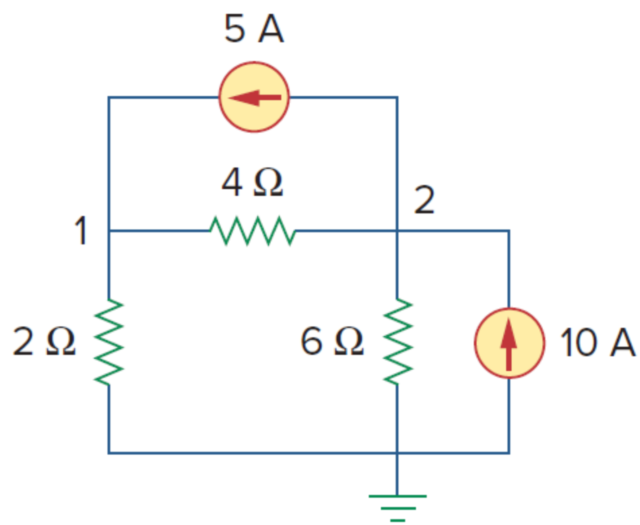
Apply KCL at node 1 and 2



$V_1 = 30\text{ V}, V_2 = -2.5\text{ V}$

3.2 Nodal Analysis (4)

Ex.2 Calculate the node voltages in the circuit



At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \quad \Rightarrow \quad 3v_1 - v_2 = 20$$

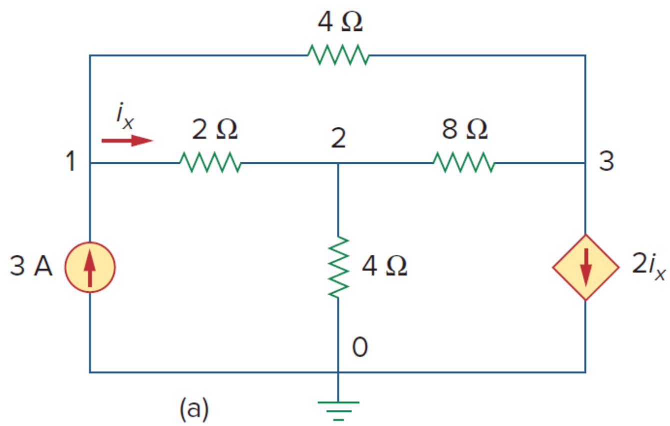
At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6} \quad \Rightarrow \quad -3v_1 + 5v_2 = 60$$

$v_1 = 13.33$ V, $v_2 = 20$ V

3.2 Nodal Analysis (5)

Ex.3 Determine the voltages at the nodes in Fig.



(a)

At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \quad \Rightarrow \quad 3v_1 - 2v_2 - v_3 = 12$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \quad \Rightarrow \quad -4v_1 + 7v_2 - v_3 = 0$$

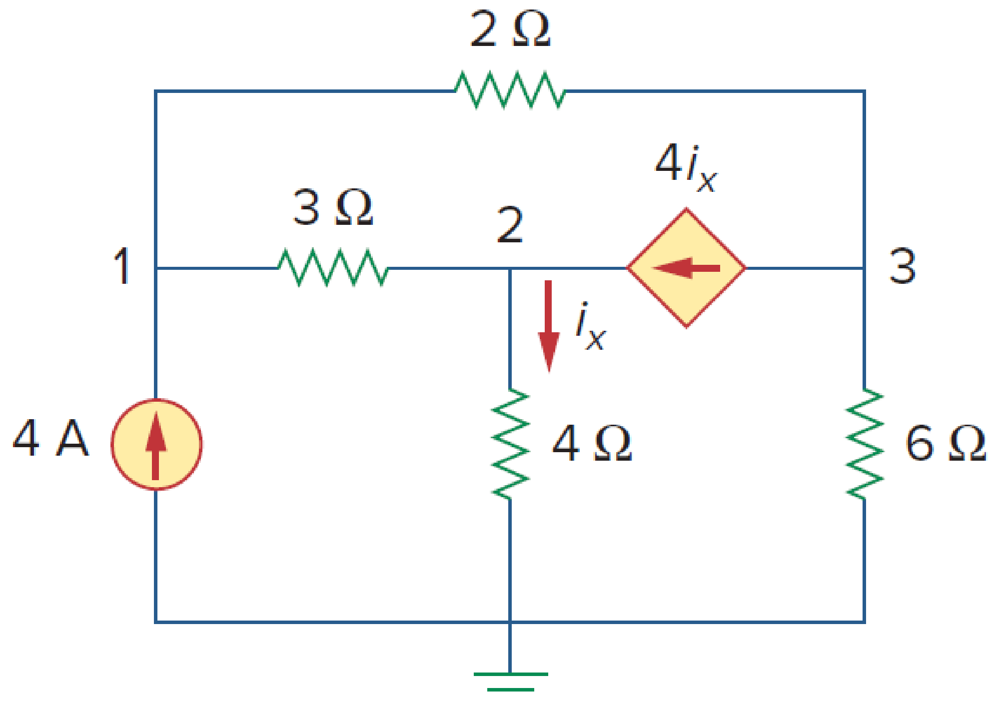
At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2} \quad \Rightarrow \quad 2v_1 - 3v_2 + v_3 = 0$$

$v_1 = 4.8\text{V}$, $v_2 = 2.4\text{V}$, $v_3 = -2.4\text{V}$

Ex.4 Find the voltages at the three non-reference nodes in the circuit of Fig.

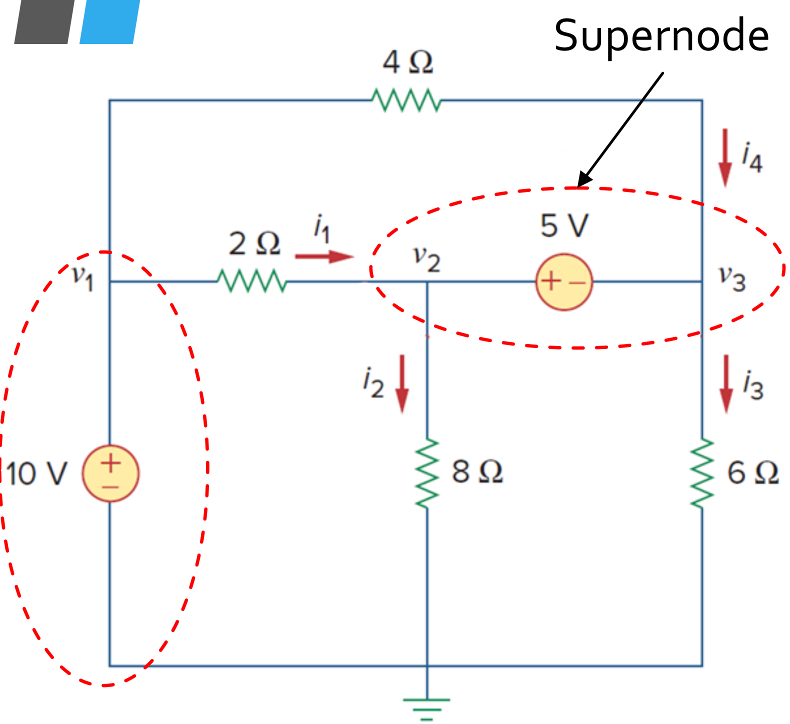
3.2 Nodal Analysis (6)



$v_1 = 32V$, $v_2 = -25.6V$, $v_3 = 62.4V$

3.3 Nodal Analysis with Voltage Sources (1)

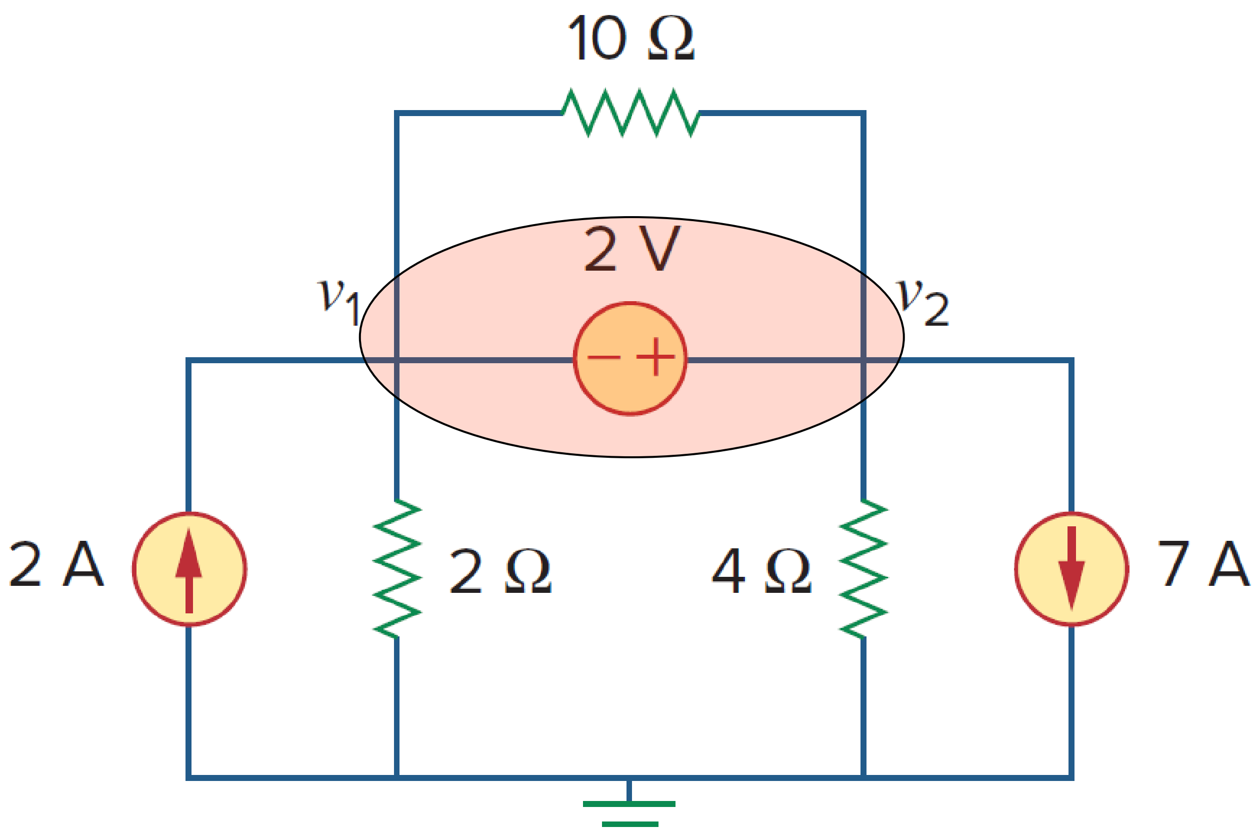
How voltage sources affect nodal analysis?



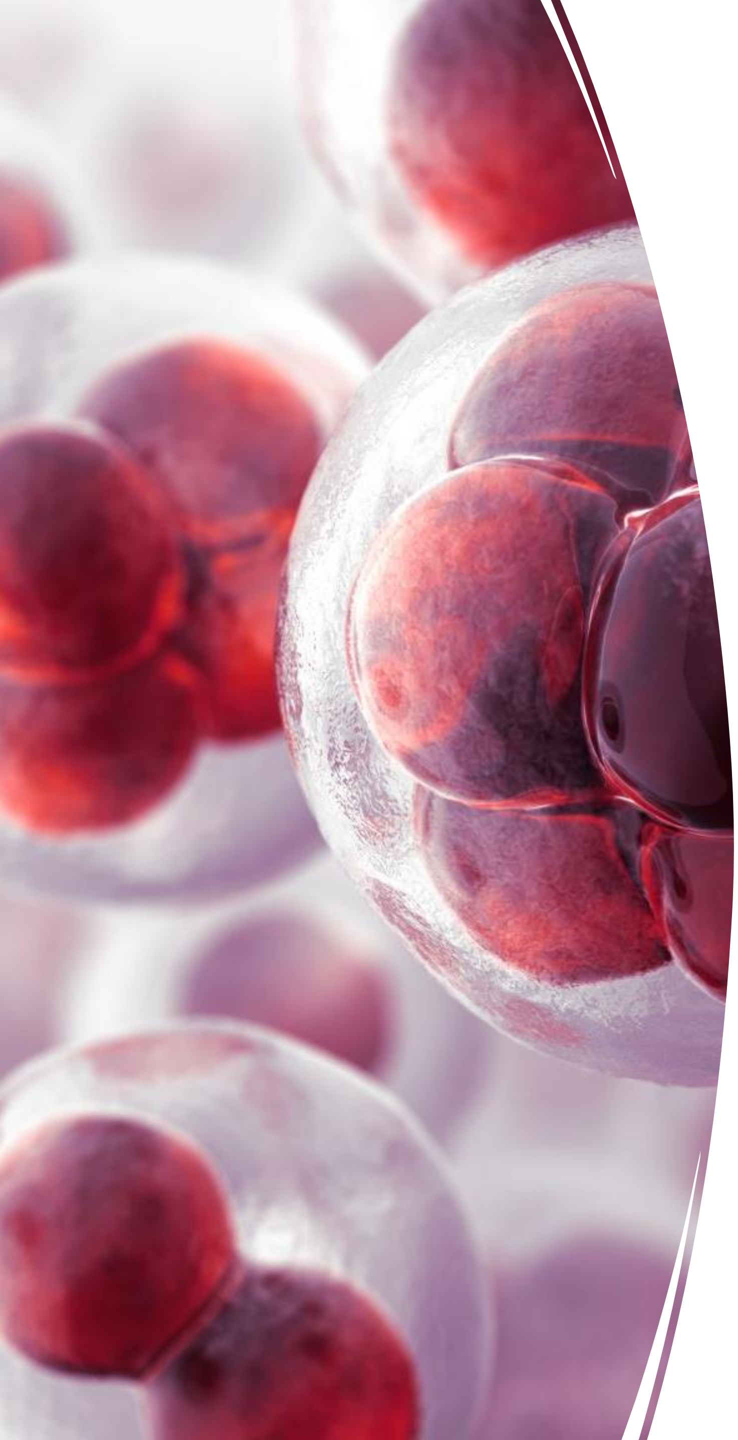
- **<u>CASE 1:</u>** If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. for example, $v_1 = 10$ V
- **<u>CASE 2:</u>** If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages. In Fig. , nodes 2 and 3 form a supernode.

$v_1 = 10\text{V},\ v_2 = 9.2\text{V},\ v_3 = 4.2\text{V}$

3.3 Nodal Analysis with Voltage Sources (2)



How to handle the 2V voltage source with Nodal Analysis?



3.3 Nodal Analysis with Voltage Sources (3)

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it. (ซูเปอร์โหนดคือโหนดที่ล้อมรอบแหล่งกำเนิดที่เชื่อมระหว่างโหนด 2 โหนด)

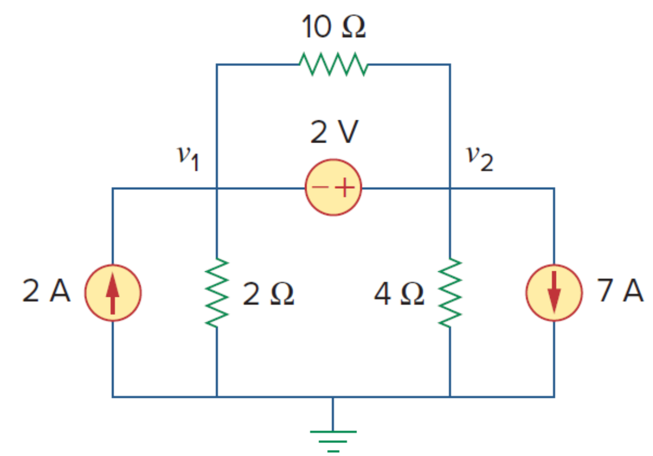
- Note: We analyze a circuit with the supernode using the same three steps mentioned above except that the supernode are treated differently.

3.3 Nodal Analysis with Voltage Sources (4)

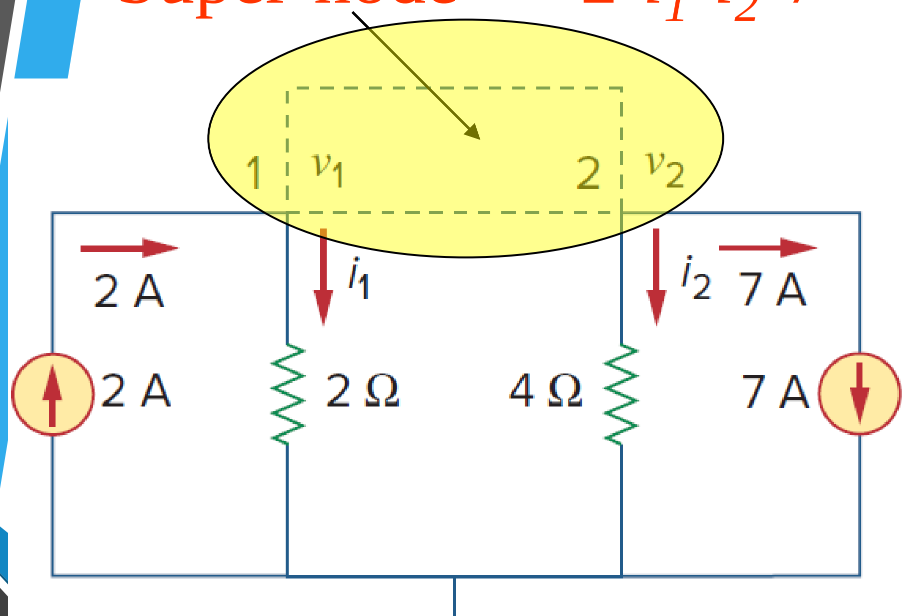
- Note: the properties of a supernode.
- The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
- A supernode has no voltage of its own.
- A supernode requires the application of both KCL and KVL.

3.3 Nodal Analysis with Voltage Sources (5)

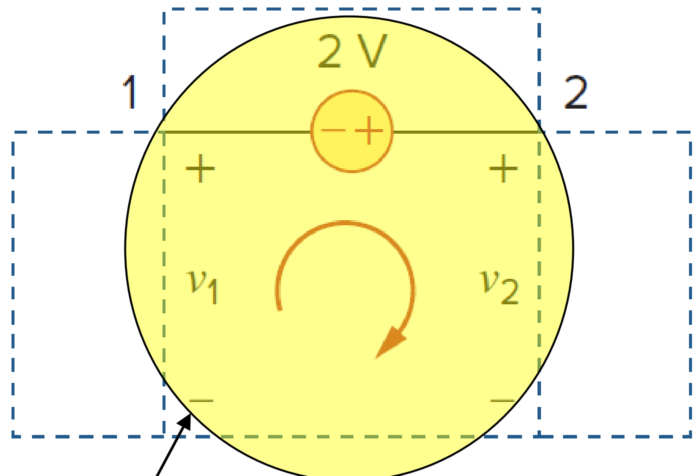
Ex.5 Find the node voltages.



Super-node => $2 - i_1 - i_2 - 7 = 0$



(a)



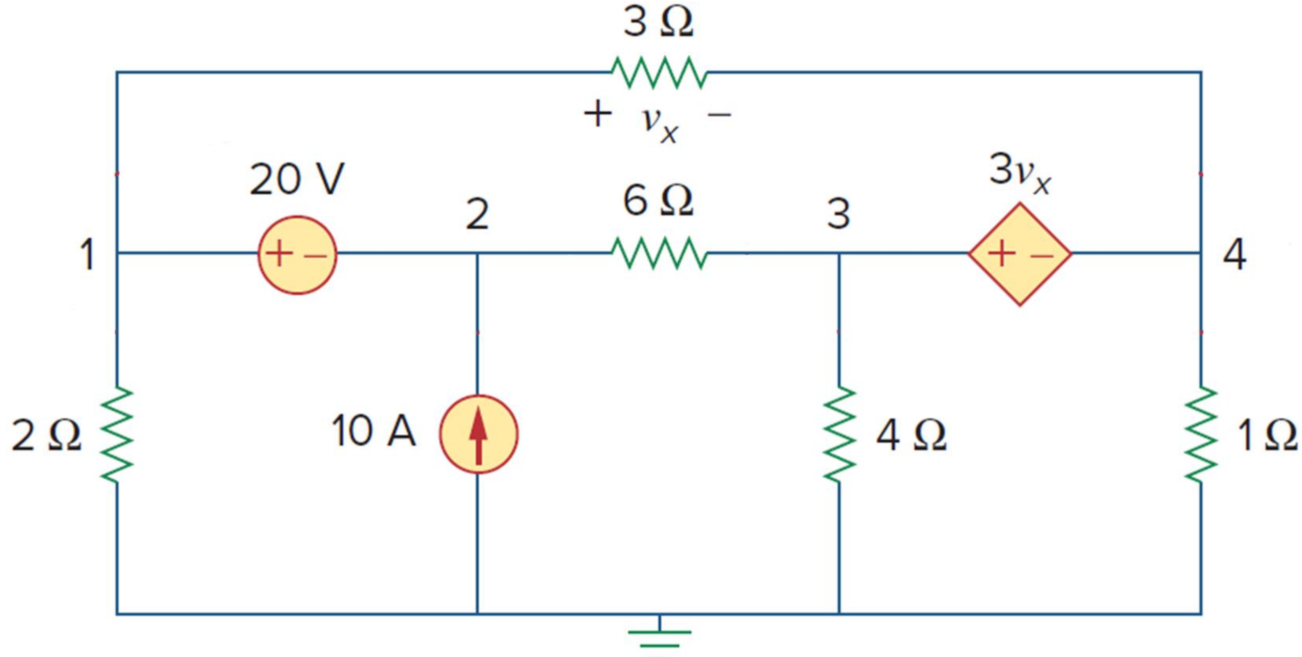
(b)

Apply KVL => $v_2 - v_1 - 2 = 0$

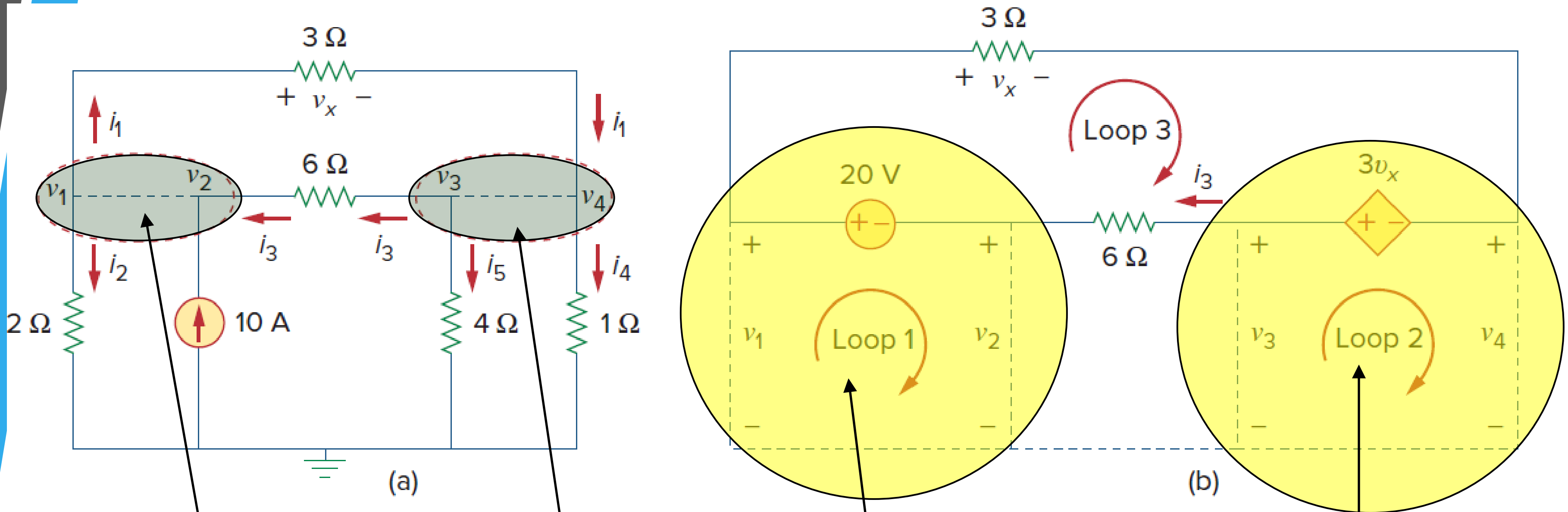
$v_1 = -7.333\text{V}$, $v_2 = -5.333\text{V}$

3.3 Nodal Analysis with Voltage Sources (6)

Ex.6 Find the node voltages in the circuit of Fig.



3.3 Nodal Analysis with Voltage Sources (7)



$$-i_1 - i_2 + i_3 + 10 = 0$$

$$-i_3 - i_5 - i_4 + i_1 = 0$$

$$v_1 - 20 - v_2 = 0$$

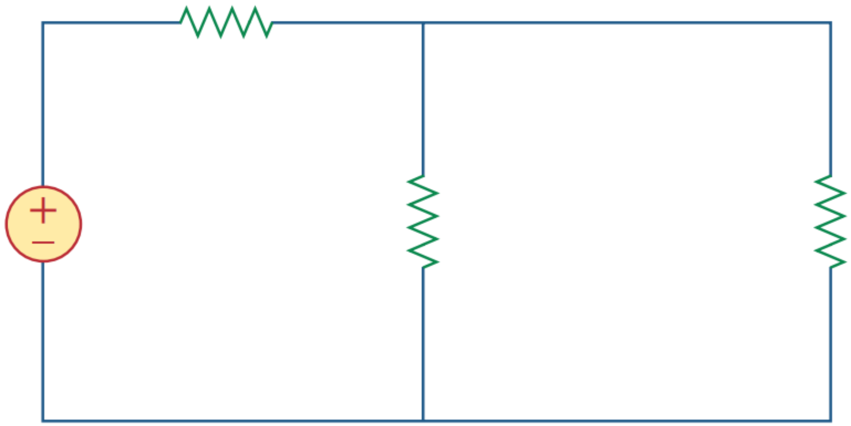
$$v_3 - 3v_x - v_4 = 0$$

$v_1 = 26.67\text{V}$, $v_2 = 6.667\text{V}$, $v_3 = 173.33\text{V}$, $v_4 = -46.67\text{V}$

3.4 Mesh Analysis (1)

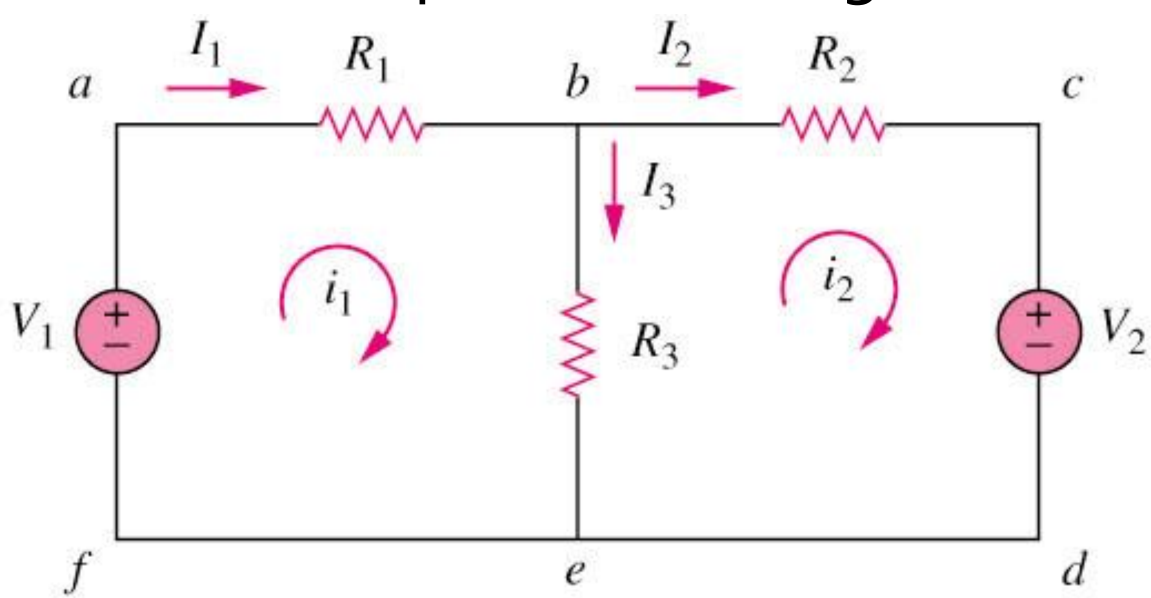
Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.



3.4 Mesh Analysis (2)

Ex.7 Circuit with independent voltage sources



Note:

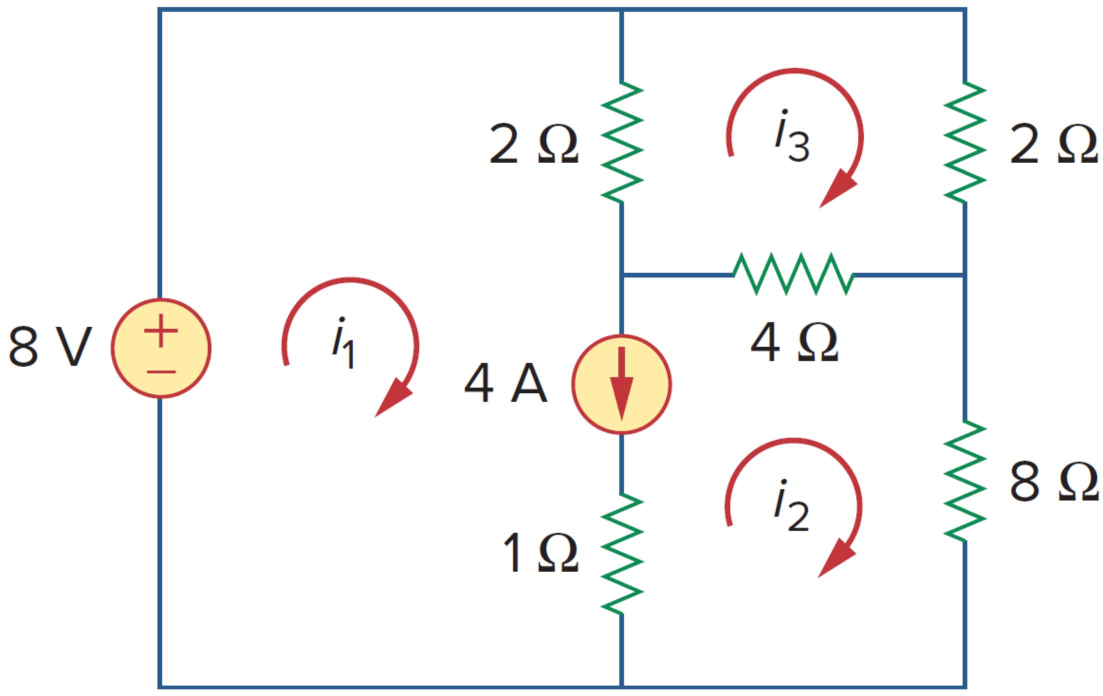
i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

3.4 Mesh Analysis (3)

<u>Ex.8</u> Use mesh analysis to determine i_1, i_2, and i_3



$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & -4 & 8 \\ 2 & 12 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$$

i_1 = 4.632 A, i_2 = 631.6 mA, i_3 = 1.4736 A.

3.5 Mesh Analysis with Current Source (1)

How current sources affect mesh analysis?

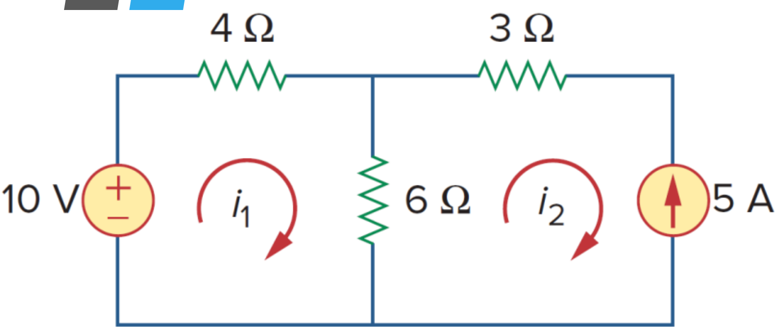


Fig.1

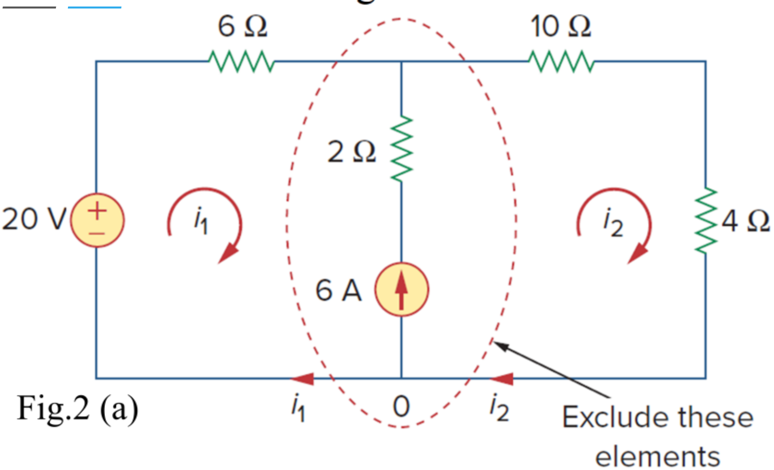


Fig.2 (a)

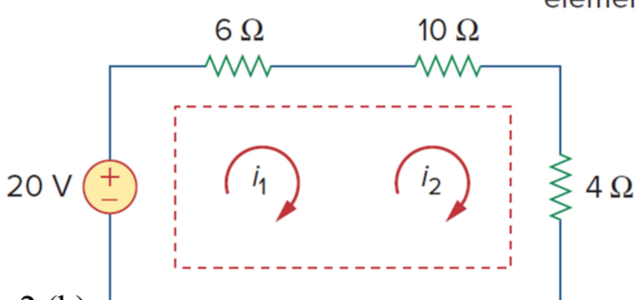


Fig.2 (b)

- Consider the following two possibilities.
- **CASE 1:** When a current source exists only in one mesh: In Fig.1, for example, we set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way; that is, $-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2$ A
- **CASE 2:** When a current source exists between two meshes: Consider the circuit in Fig. (a), for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. (b). Thus,
- $-20 + 6i_1 + 10i_2 + 4i_2 = 0 \Rightarrow 6i_1 + 14i_2 = 20$
- Applying KCL to node 0 in Fig. (a) gives
- $i_2 = i_1 + 6$
- Solving both Eqs. above/.
- $i_1 = -3.2$ A, $i_2 = 2.8$ A

3.5 Mesh Analysis with Current Source (2)

<u>The properties of a supermesh:</u>

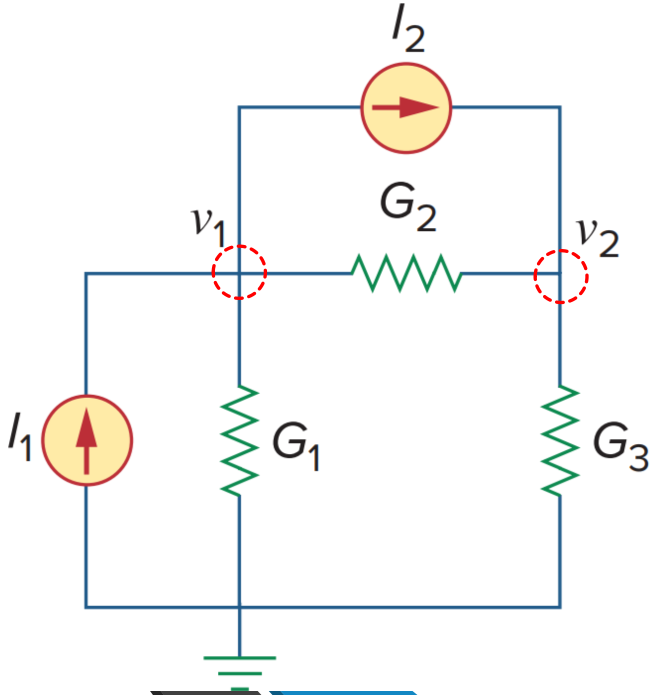
- ❑ The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
- ❑ A supermesh has no current of its own.
- ❑ A supermesh requires the application of both KVL and KCL.

3.6 Nodal Analysis by Inspection (1)

Each of the diagonal terms is the sum of the conductances connected directly to node 1 or 2,

while the off-diagonal terms are the negatives of the conductances connected between the nodes.

Each term on the right-hand side is the algebraic sum of the currents entering the node.



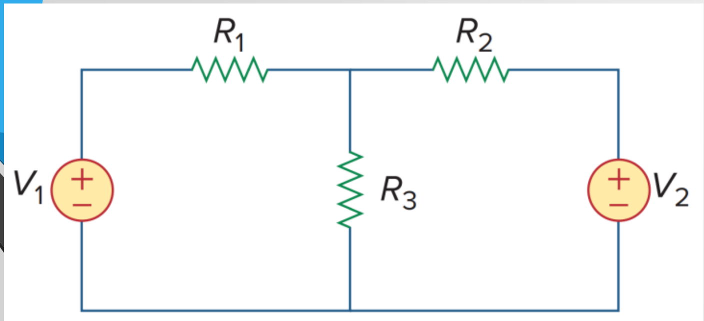
$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

3.6 Mesh Analysis by Inspection (2)

Each of the diagonal terms is the sum of the resistances in the related mesh,

while each of the off-diagonal terms is the negative of the resistance common to meshes 1 and 2.

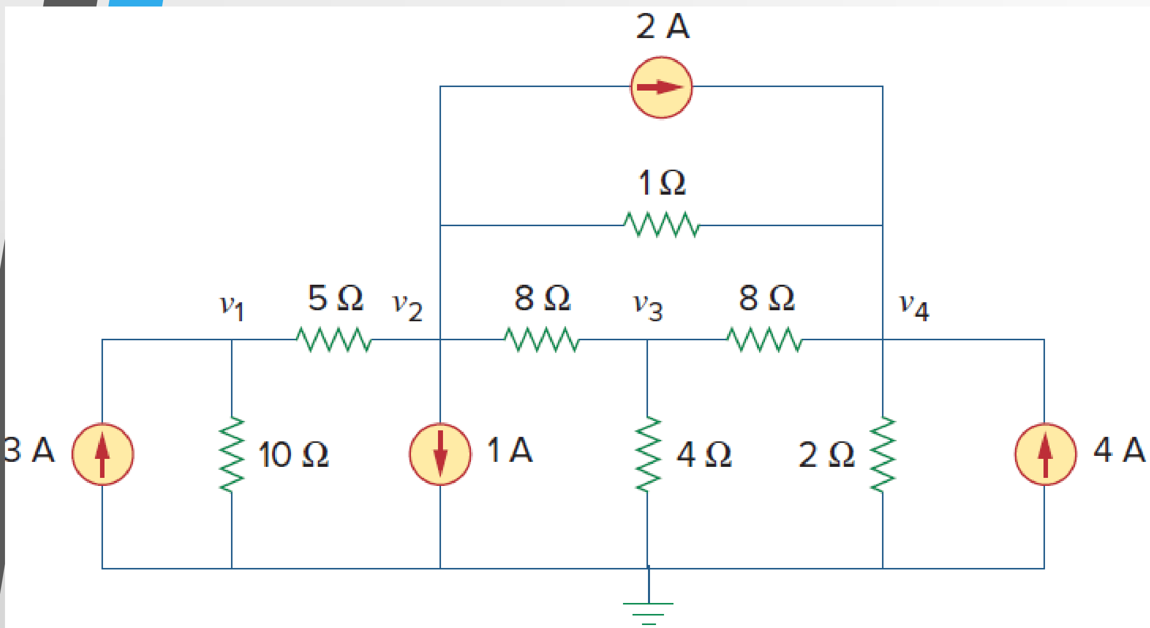
Each term on the right-hand side of Eq. (3.24) is the algebraic sum taken clockwise of all independent voltage sources in the related mesh.



$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

3.6 Node Analysis by Inspection (3)

Ex.9 Write the nodal voltage equations for the circuit



$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \qquad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \qquad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \quad G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \qquad G_{32} = -0.125, \qquad G_{34} = -\frac{1}{8} = -0.125$$

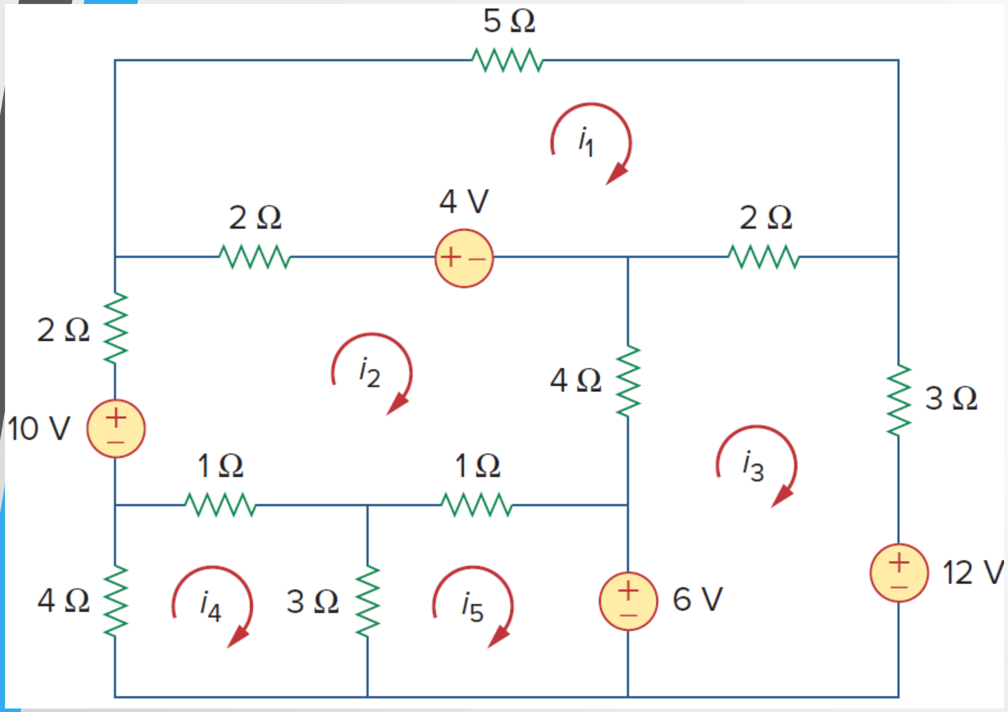
$$G_{41} = 0, \qquad G_{42} = -1, \qquad G_{43} = -0.125$$

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

v_1 = 13.897 V, v_2 = 5.845 V, v_3 = 3.348 V , v_4 = 7.547 V.

3.6 Mesh Analysis by Inspection (4)

Ex.10 Write the mesh-current equations for the circuit

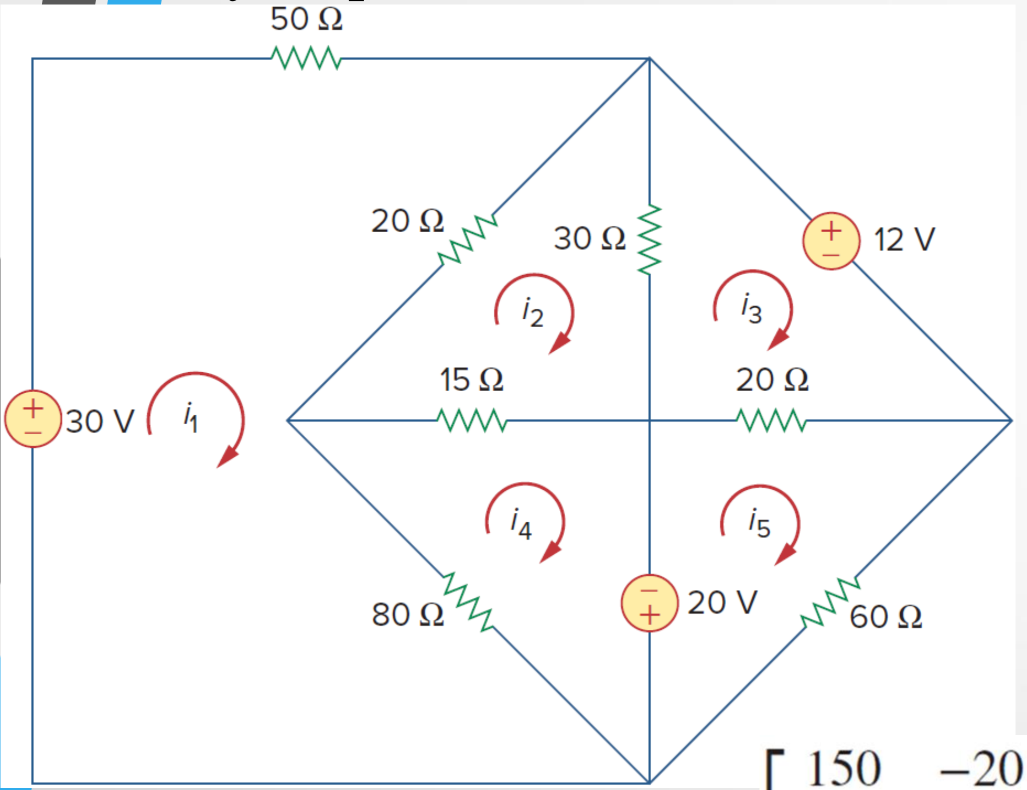


$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

i_1 = 0.3829 A, i_2 = 0.2110 A, i_3 = -0.4878 A, i_4 = -0.7184 A, i_5 = -1.986 A.

3.6 Mesh Analysis by Inspection (5)

Ex.11 By inspection, obtain the mesh-current equations for the circuit



$$\begin{bmatrix} 150 & -20 & 0 & -80 & 0 \\ -20 & 65 & -30 & -15 & 0 \\ -0 & -30 & 50 & 0 & -20 \\ -80 & -15 & 0 & 95 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix}$$

i_1 = 0.7039 A, i_2 = 0.3470 A, i_3 = -0.1464 A, i_4 = 0.8581 A, i_5 = -0.2866 A.

3.7 Nodal vs Mesh Analysis

➢ Which method is better or more efficient?

1. *Choose *nodal analysis* for circuit with fewer nodes than meshes.

   *Choose *mesh analysis* for circuit with fewer meshes than nodes.

   *Networks with parallel-connected elements, current sources, or super-nodes are more suitable for *nodal analysis*.

   *Networks that contain many series connected elements, voltage sources, or super-meshes are more suitable for *mesh analysis.*

2. If node voltages are required, applying *nodal analysis*.

   If branch or mesh currents are required, applying *mesh analysis.*

Be familiar with both methods of analysis