Electrical Engineering 1
12026105

Chapter 13

Magnetically Coupled Circuits

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Understand the physics behind mutually coupled circuits and how to analyze circuits containing mutually coupled inductors.
2. Understand how energy is stored in mutually coupled circuits.
3. Understand how linear transformers work and how to analyze circuits containing them.
4. Understand how ideal transformers work and how to analyze circuits containing them.
5. Understand how ideal auto transformers work and know how to analyze them when used in a variety of circuits.

วัตถุประสงค์การเรียนรู้

โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:

1. เข้าใจฟิสิกส์เบื้องหลังวงจรเชื่อมต่อทางแม่เหล็กและวิธีวิเคราะห์วงจรเชื่อมต่อทางแม่เหล็ก
2. เข้าใจว่าพลังงานถูกเก็บไว้ในวงจรเชื่อมต่อทางแม่เหล็ก
3. เข้าใจว่าหม้อแปลงเชิงเส้นทำงานอย่างไรและสามารถวิเคราะห์วงจรไฟฟ้าที่มีหม้อแปลงเชิงเส้นได้
4. เข้าใจว่าหม้อแปลงในอุดมคติทำงานอย่างไรและสามารถวิเคราะห์วงจรไฟฟ้าที่มีหม้อแปลงในอุดมคติได้
5. เข้าใจว่าหม้อแปลงไฟฟ้าอัตโนมัติในอุดมคติทำงานอย่างไรและสามารถวิเคราะห์วงจรไฟฟ้าที่มีหม้อแปลงไฟฟ้าอัตโนมัติในอุดมคติได้

Magnetically Coupled Circuit Chapter 13

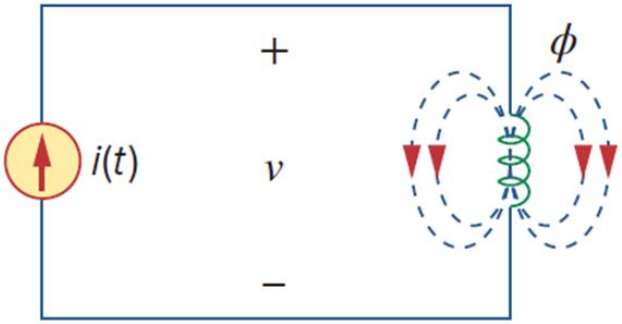
- What is a transformer?
- Mutual Inductance
- Energy in a Coupled Circuit
- Linear Transformers
- Ideal Transformers
- Applications

13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling (การเชื่อมต่อทางแม่เหล็ก)
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down AC voltages or currents, impedance matching, isolation, etc.

13.2 Mutual Inductance (1)

- A single inductor, a coil with N turns. When current i flows through the coil, a magnetic flux ϕ is produced around it.



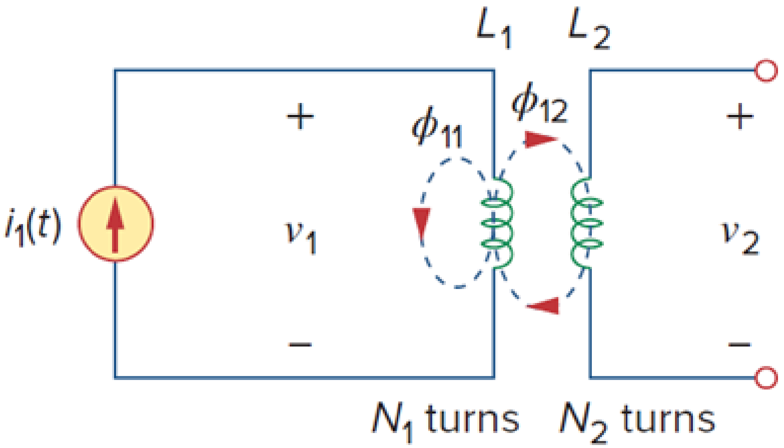
Magnetic flux produced by a single coil with N turns.

- According to Faraday's law, $v = N\dfrac{d\phi}{dt}$
- The flux ϕ is produced by current i so that any change in ϕ is caused by a change in the current i.

$$v = N\frac{d\phi}{di}\frac{di}{dt} \Rightarrow v = L\frac{di}{dt} \Rightarrow L = N\frac{d\phi}{di}$$

L is commonly called *self-inductance*

13.2 Mutual Inductance (2)



Two coils close proximity with each other. Coil 1 has N_1 turns, while coil 2 has N_2 turns. For the sake of simplicity, assume that *the second inductor carries no current*.

- The magnetic flux ϕ_1 emanating from coil 1 has two components: One component ϕ_{11} links only coil 1, and another component ϕ_{12} links both coils. Hence, $\phi_1 = \phi_{11} + \phi_{12}$
- Although the two coils are physically separated, they are *magnetically coupled*. Since the entire flux ϕ_1 links coil 1, the voltage induced in coil 1 is $v_1 = N_1 \dfrac{d\phi_1}{dt}$
- Only flux ϕ_{12} links coil 2, so the voltage induced in coil 2 is $v_2 = N_2 \dfrac{d\phi_{12}}{dt}$

13.2 Mutual Inductance (3)

- Again, as the fluxes are caused by the current i_1 flowing in coil 1, so

  that $v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$,where L_1 is the self-inductance of coil 1.

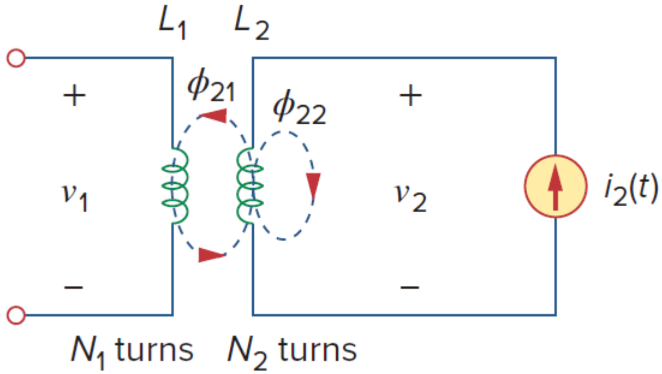
- And $v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$,Where $M_{21} = N_2 \frac{d\phi_{12}}{di_1}$ is the mutual

  inductance of coil 2 wrt. coil 1.

- Thus, the open-circuit mutual voltage (or induced voltage) across coil

  2 is $$v_2 = M_{21}\frac{di_1}{dt}$$

13.2 Mutual Inductance (4)



Suppose now let current i_2 flow in coil 2, while coil 1 carries no current.

- The magnetic flux ϕ_2 emanating from coil 2 has two components: One component ϕ_{22} links only coil 2, and another component ϕ_{21} links both coils. Hence, $\phi_2 = \phi_{21} + \phi_{22}$
- The entire flux ϕ_2 links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

- Since only flux ϕ_{21} links coil 1, the voltage induced in coil 1 is ,

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

,Where $M_{12} = N_1 \frac{d\phi_{21}}{di_2}$ is the mutual inductance of coil 1 wrt. coil 2.

13.2 Mutual Inductance (5)

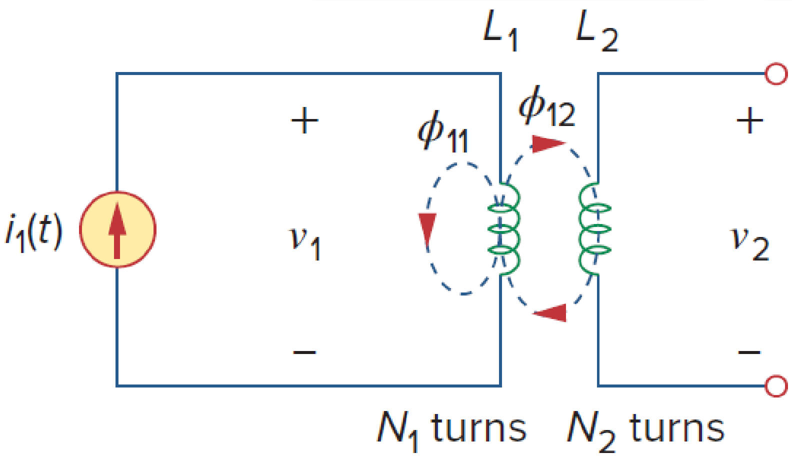
- Thus, the open-circuit mutual voltage across coil 1 $v_1 = M_{12} \frac{di_2}{dt}$
- M_{12} and M_{21} are equal or $M_{12} = M_{21} = M$

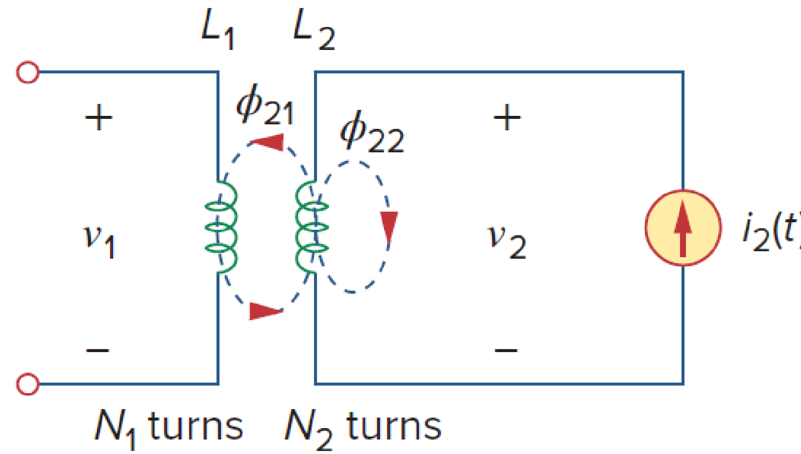
Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

- Mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources.
- Recall that inductors act like short circuits to dc.

13.2 Mutual Inductance (6)

- It is the ability of one inductor to induce a voltage across a neighboring inductor, measured in Henrys (H).





$$v_2 = M_{21}\frac{di_1}{dt}$$

$$M_{21} = N_2\frac{d\phi_{12}}{di_1}$$

mutual inductance of coil 2 with respect to coil 1

The open-circuit mutual voltage across coil 2

$$v_1 = M_{12}\frac{di_2}{dt}$$

$$M_{12} = N_1\frac{d\phi_{21}}{di_2}$$

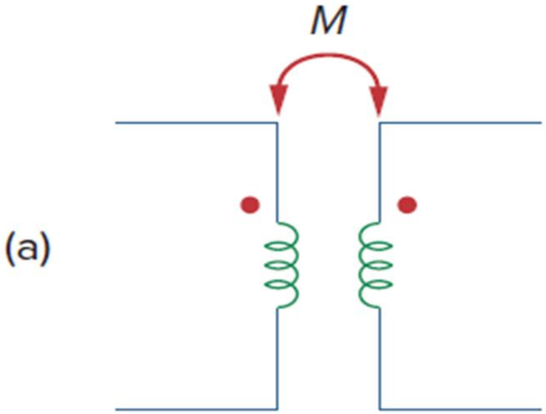
mutual inductance of coil 1 with respect to coil 2

The open-circuit mutual voltage across coil 1

13.2 Mutual Inductance (7)

- Although mutual inductance M is always a positive quantity, the mutual voltage $M\, di/dt$ may be negative (-) or positive (+).
- The polarity of mutual voltage $M\, di/dt$ is not easy to determine, because four terminals are involved.
- Applying the dot convention (•) in circuit analysis, a dot is placed in the circuit at one end of each of the two magnetically coupled coils.

13.2 Mutual Inductance (8/1)



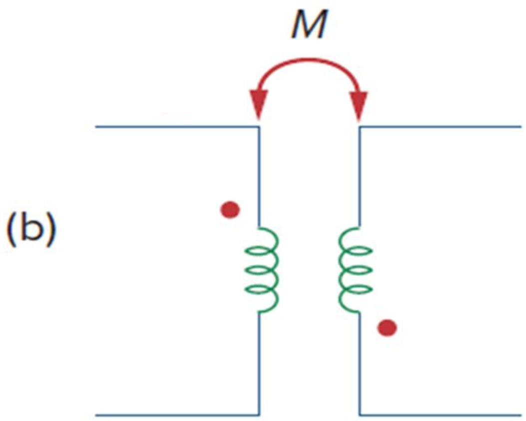
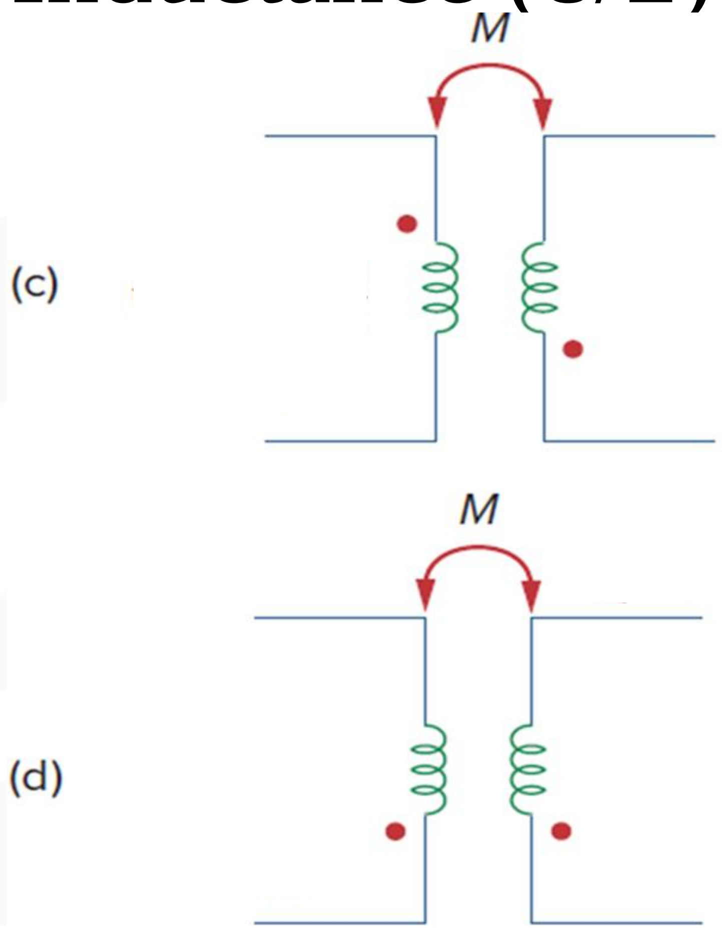


Fig.(a), the sign of the mutual voltage v_2 is determined by the reference polarity for v_2 and the direction of i_1. Since i_1 enters the dotted terminal of coil 1 and v_2 is positive at the dotted terminal of coil 2, the mutual voltage is $M\frac{di_1}{dt}$

Fig.(b), the current i_1 enters the dotted terminal of coil 1 and v_2 is negative at the dotted terminal of coil 2. Hence, the mutual voltage is $-M\frac{di_1}{dt}$

13.2 Mutual Inductance (8/2)



13.2 Mutual Inductance (9)

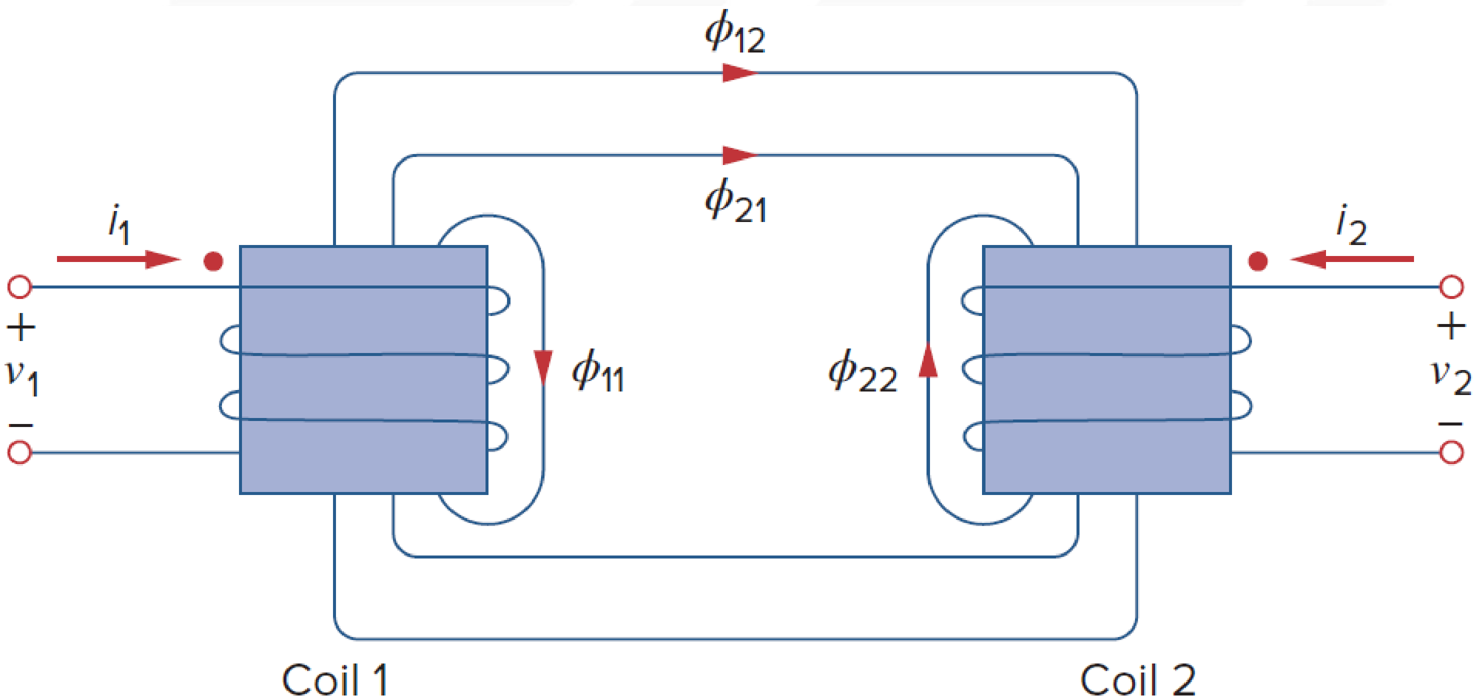
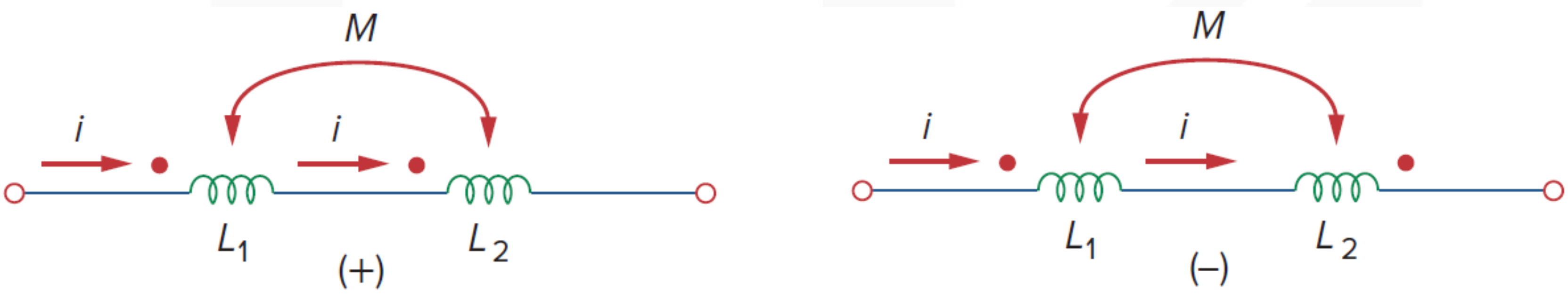


Illustration of the dot convention.

13.2 Mutual Inductance (10)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage;

(a) series-aiding connection, (b) series-opposing connection.



(a) (b)

$$L = L_1 + L_2 + 2M$$

(Series-aiding connection)

$$L = L_1 + L_2 - 2M$$

(Series-opposing connection)

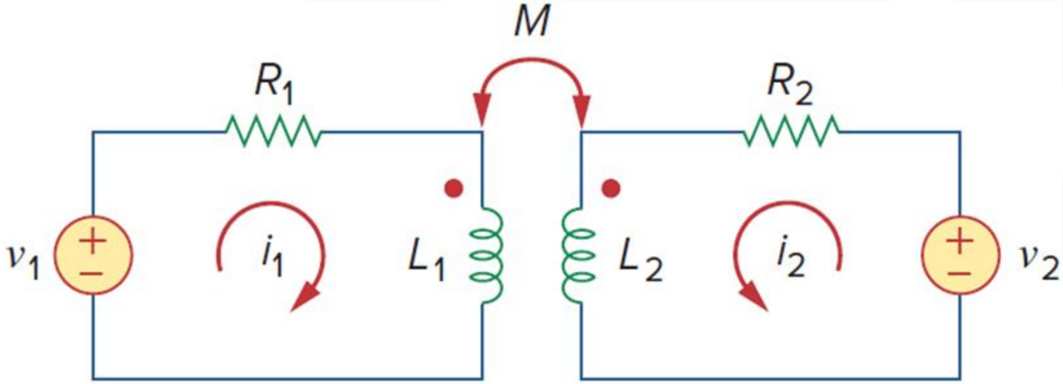
13.2 Mutual Inductance (11)

Considering Fig., applying KVL to coil 1 gives $$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

For coil 2, applying KVL gives $$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

We can write Eq. above in the frequency domain as $$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$



- Time-domain analysis of a circuit containing coupled coils.

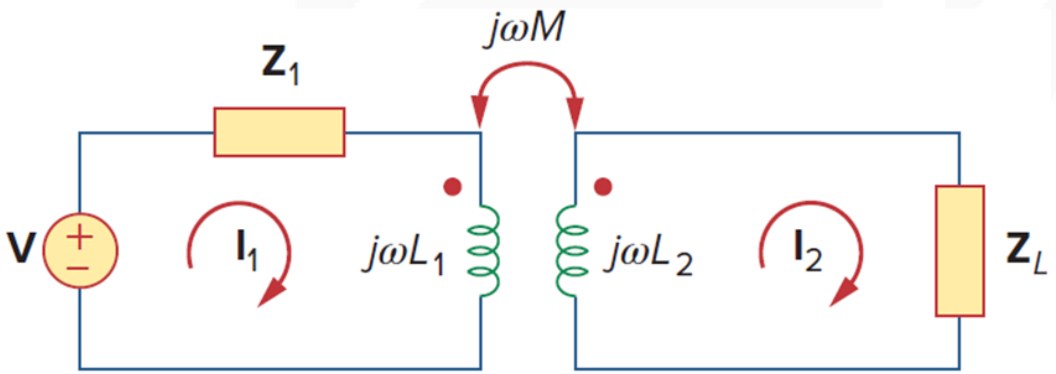
13.2 Mutual Inductance (12)

Considering the circuit in Fig.. We analyze this in the frequency domain,

Applying KVL to coil 1 gives $\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$

For coil 2, KVL yields $0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$

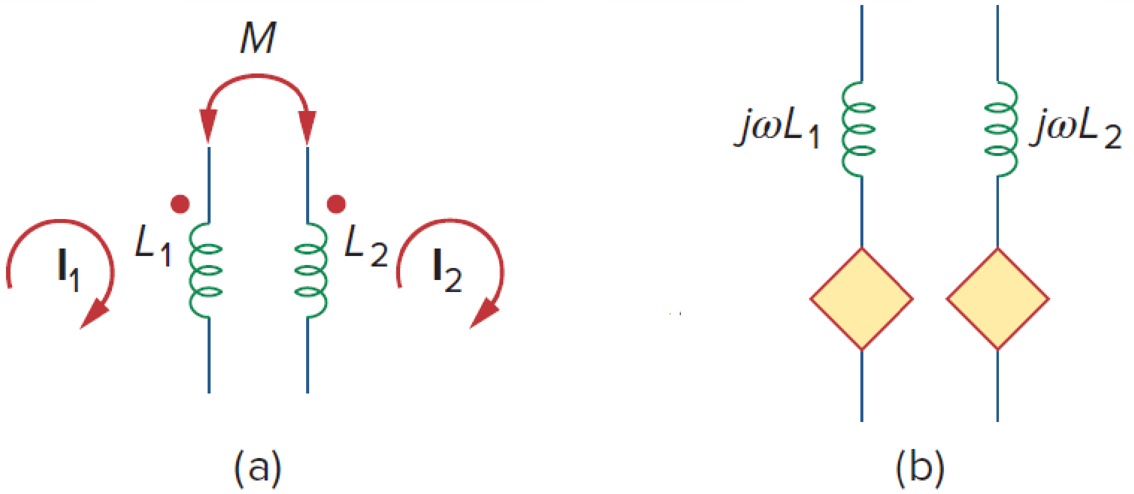
Equations are solved in the usual manner to determine the currents.



- Frequency-domain analysis of a circuit containing coupled coils

13.2 Mutual Inductance (13)

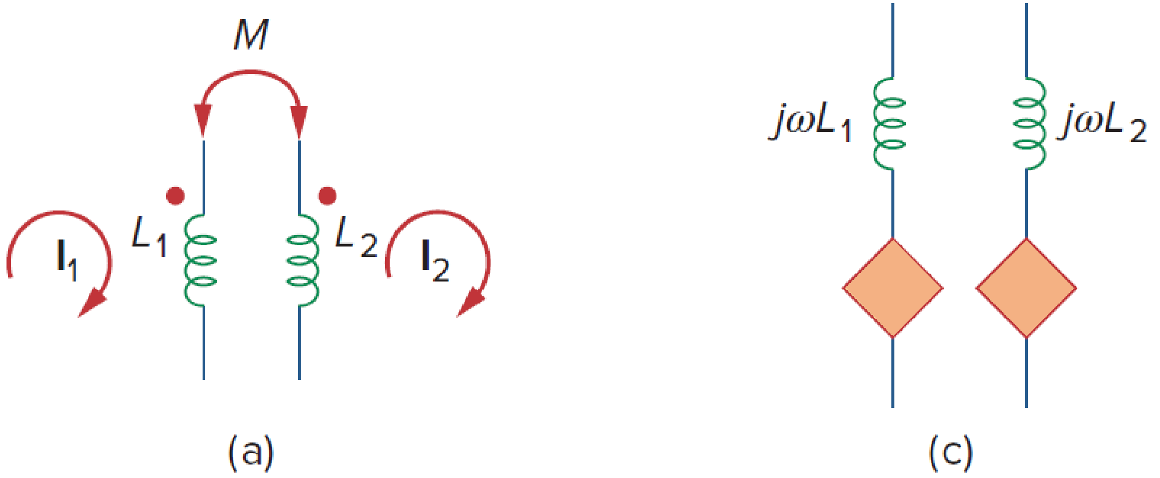
- Breaking the problem into steps of solving for the value and the sign into separate steps
- Suggest that model (Figure (b)) be used when analyzing circuits containing a mutually coupled circuit shown in Figure (a):
- Clearly, I_1 induces a voltage within the second coil represented by the value $j\omega M I_1$ and I_2 induces a voltage of $j\omega M I_2$ in the first coil.



Model that makes analysis of mutually coupled easier to solve.

13.2 Mutual Inductance (14)

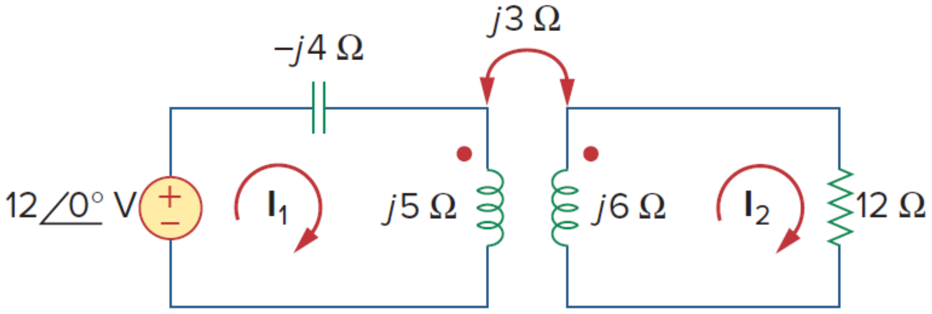
- Find the correct signs for the dependent sources as shown in Figure 13.8(c).
- Since I_1 enters L_1 at the dotted end, it induces a voltage in L_2 which means that the source must have a plus on top and a minus on the bottom as shown in Figure (c).
- I_2 leaves the dotted end of L_2 which means that it induces a voltage in L_1 which has a plus on the bottom and a minus on top as shown in Figure (c).
- Now all we have to do is to analyze a circuit with two dependent sources.



Model that makes analysis of mutually coupled easier to solve.

13.2 Mutual Inductance (15)

<u>Ex.1</u> Calculate the phasor currents I_1 and I_2 in the circuit shown below.



(a)

Solution:

For loop 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For loop 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$$

Substituting this in Eq., we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

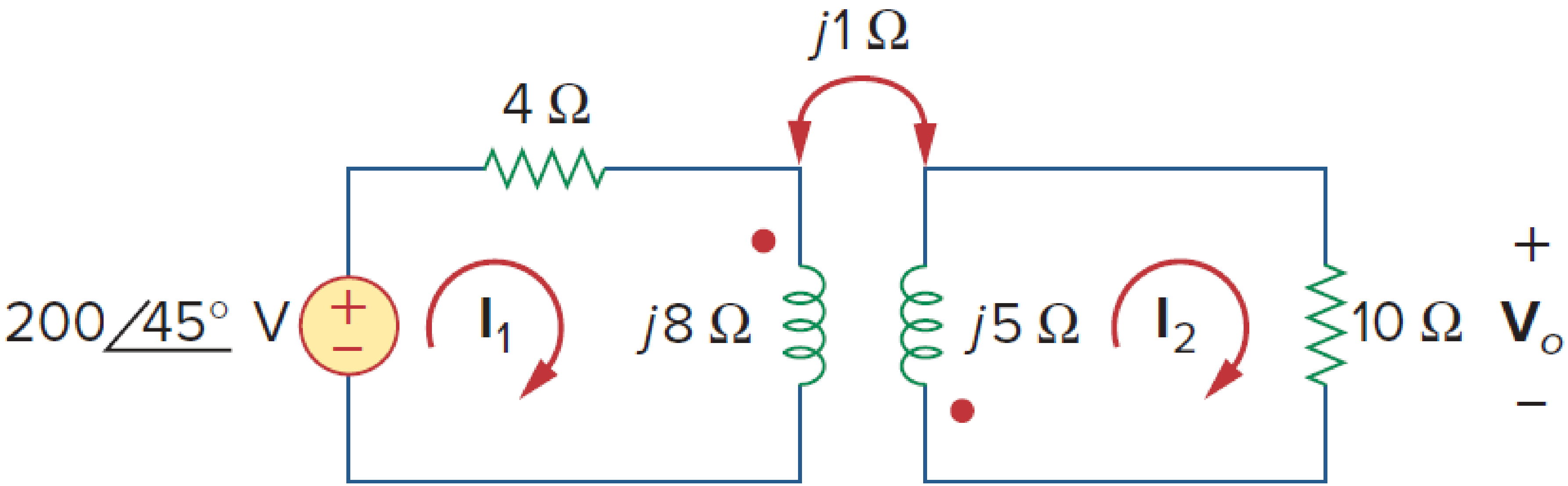
or

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ}\text{ A}$$

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$
$$= 13.01\underline{/-49.39^\circ}\text{ A}$$

13.2 Mutual Inductance (16)

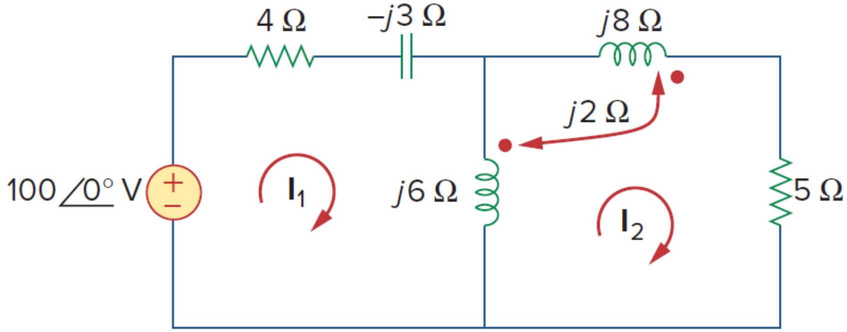
Ex.2 Determine the voltage V_o in the circuit

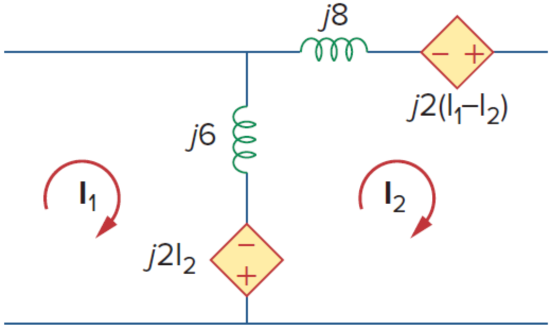


Answer: $20\angle -135^\circ$ V.

13.2 Mutual Inductance (17)

<u>Ex.3</u> Calculate the phasor currents I_1 and I_2 in the circuit shown below.





for mesh 1 in Fig., KVL gives

$$-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$

or

$$100 = (4 + j3)\mathbf{I}_1 - j8\mathbf{I}_2$$

for mesh 2 in Fig., KVL gives

$$0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$$

or

$$0 = -j8\mathbf{I}_1 + (5 + j18)\mathbf{I}_2$$

Putting Eqs. (13.2.1) and (13.2.2) in matrix form, we get

$$\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{vmatrix} = 30 + j87$$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5 + j18 \end{vmatrix} = 100(5 + j18)$$

$$\Delta_2 = \begin{vmatrix} 4 + j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

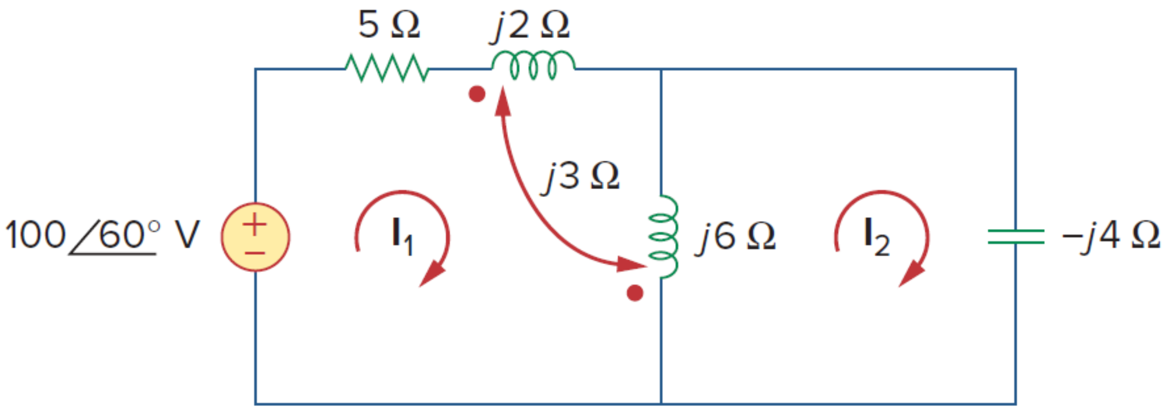
Thus, we obtain the mesh currents as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{100(5 + j18)}{30 + j87} = \frac{1{,}868.2\angle 74.5^\circ}{92.03\angle 71^\circ} = 20.3\angle 3.5^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{j800}{30 + j87} = \frac{800\angle 90^\circ}{92.03\angle 71^\circ} = 8.693\angle 19^\circ \text{ A}$$

13.2 Mutual Inductance (18)

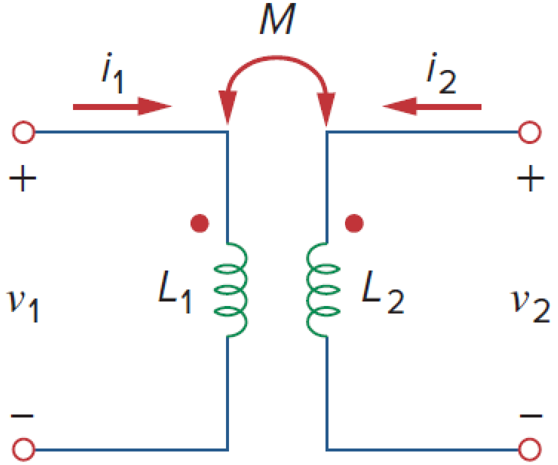
Ex.4 Determine the phasor currents circuit I_1 and I_2 in the circuit



Answer: $I_1 = 17.889\angle 86.57°$ A, $I_2 = 26.83\angle 86.57°$ A.

13.3 Energy in a Coupled Circuit (1)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \le k \le 1$.



The circuit for deriving energy stored in a coupled circuit.

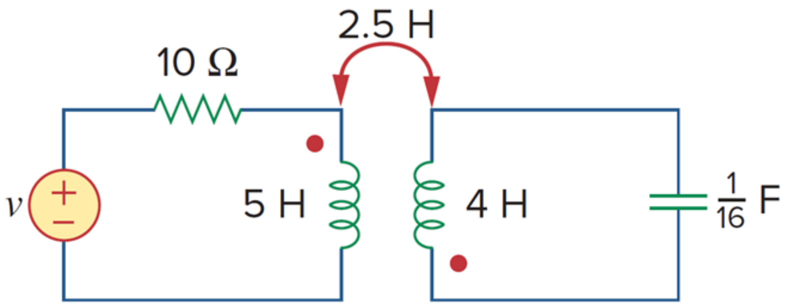
$$M = k\sqrt{L_1 L_2}$$

- The instantaneous energy stored in the circuit is given by

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M I_1 I_2$$

13.3 Energy in a Coupled Circuit (2)

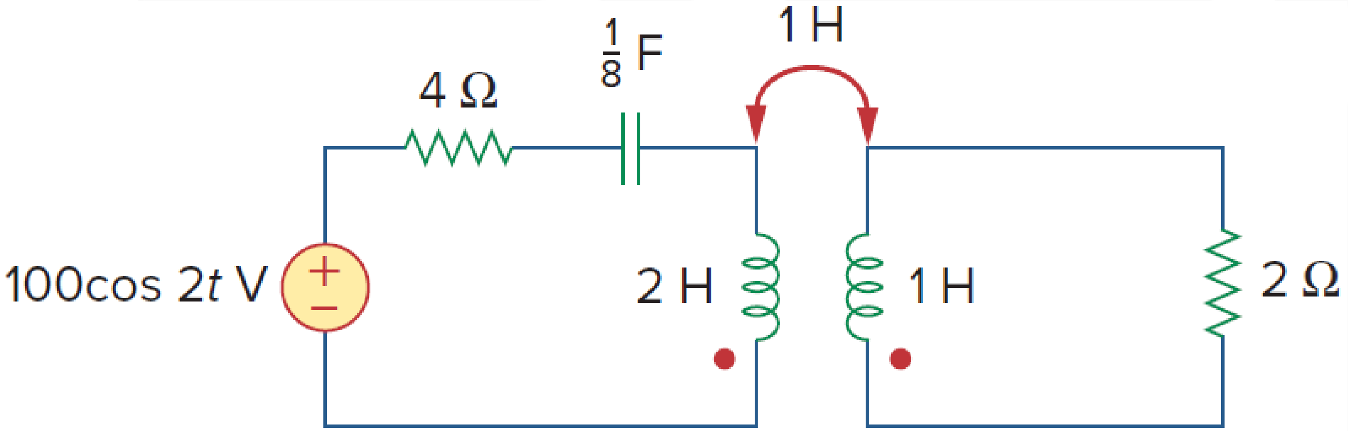
Ex.5 Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t=1s if v=60cos(4t +30°) V.



Ans: k=0.56; w(1)=20.73J

13.3 Energy in a Coupled Circuit (3)

Ex.6 Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t=1.5s

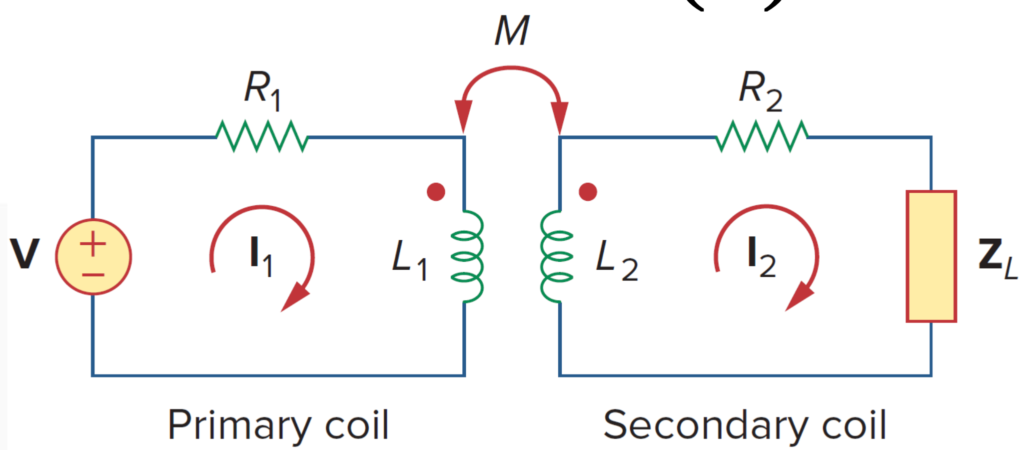


Answer: 0.7071, 246.2 J.

13.4 Linear Transformer (1)

- The transformer will be linear if the coils are wound(พัน) on a magnetically linear material—a material for which the magnetic permeability is constant. (Such materials include air, plastic, Bakelite, and wood.)
- In fact, most materials are magnetically linear. Linear transformers are sometimes called air-core transformers, although not all of them are necessarily air-core.
- It is generally a four-terminal device comprising two (or more) magnetically coupled coils.

13.4 Linear Transformer (2)



Different types of transformers:
(a) a large substation transformer, (b) audio transformers.

(a)

(b)

13.4 Linear Transformer (3)

- Applying KVL to the two meshes in Fig. gives

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 \qquad (1)$$

$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2 \qquad (2)$$

- In (2),we express $\mathbf{I}_2$ in terms of $\mathbf{I}_1$ and substitute it into (1). We get the input impedance as

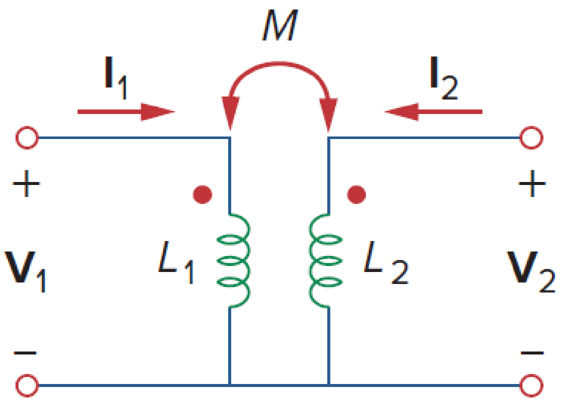
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

- The input impedance comprises 2 terms.
- The first term, $(R_1 + j\omega L_1)$, is the primary impedance.
- The second term is due to the coupling between the primary and secondary windings. It is as though this impedance is reflected to the primary. Thus, it is known as the reflected impedance Z_R, and

$$\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

13.4 Linear Transformer (4)

- The voltage-current relationships for the primary and secondary coils give the matrix equation



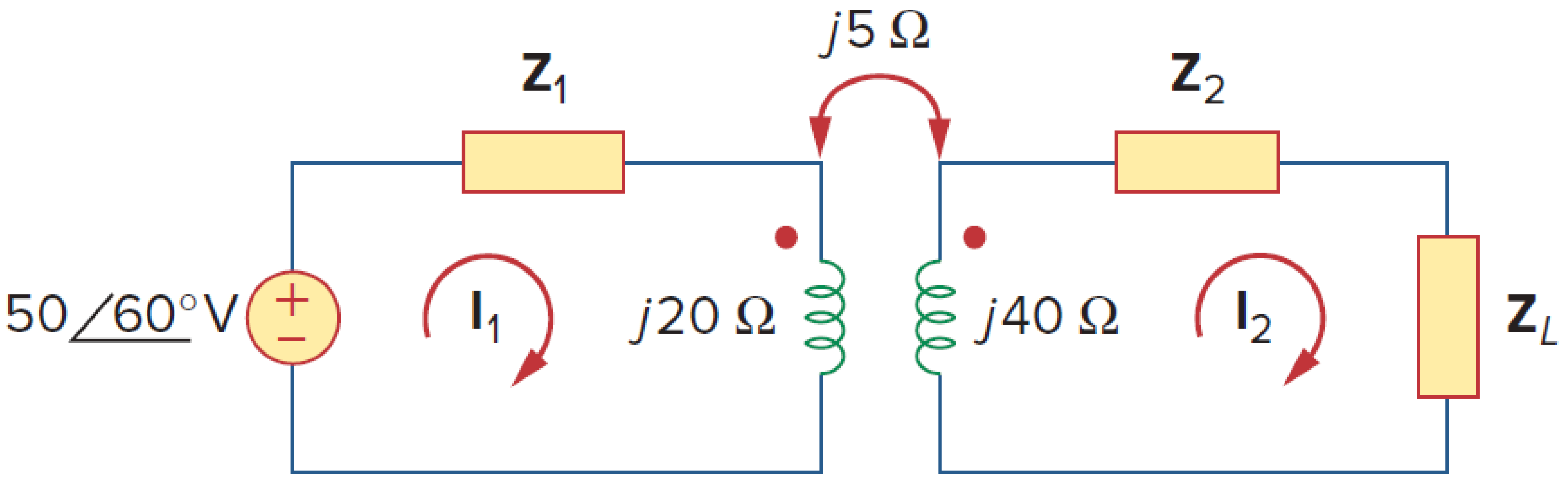
Determining the equivalent circuit of a linear transformer.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

13.4 Linear Transformer (5)

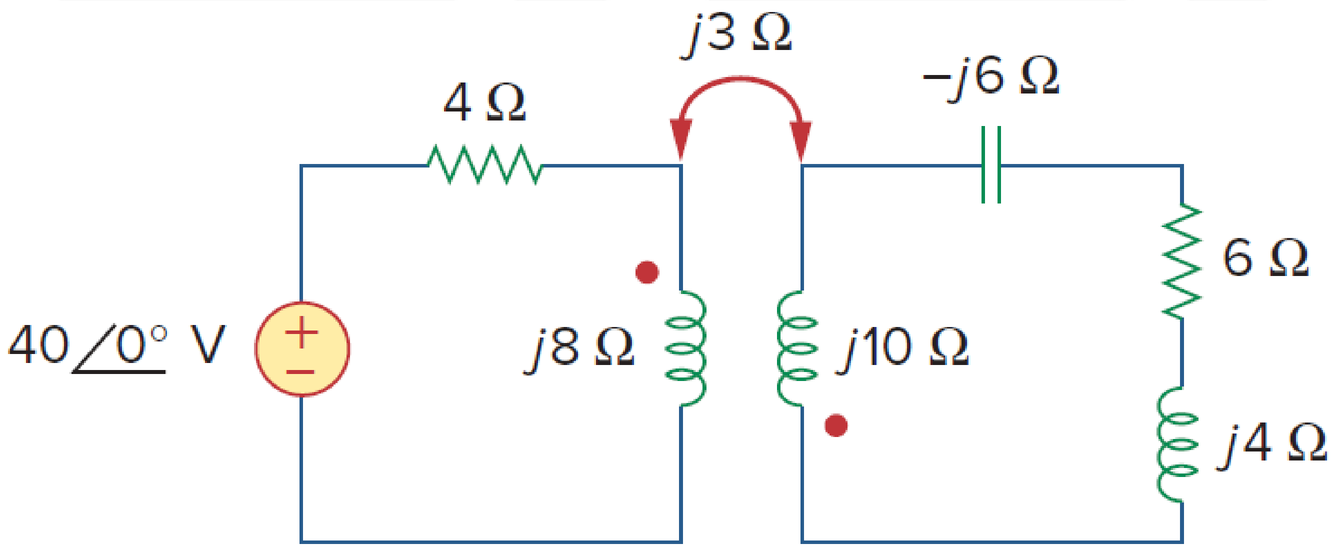
Ex.7 In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-j100Ω, Z_2=30+j40Ω, and Z_L=80+j60Ω.



$Z_{in} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°A$

13.4 Linear Transformer (6)

Ex.8 Find the input impedance of the circuit in Fig. and the current from the voltage source.



Answer: $8.58\angle 58.05^\circ\ \Omega$, $4.662\angle -58.05^\circ$ A.

13.5 Ideal Transformer (1)

An **ideal transformer** is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

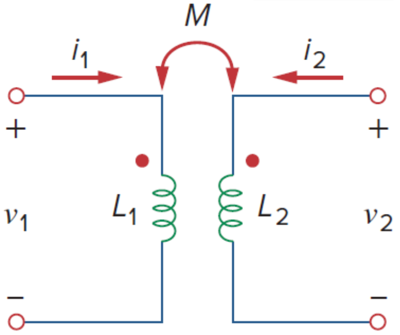
- Let us reexamine the circuit in Fig., In the frequency domain

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2 \Rightarrow \mathbf{I}_1 = (\mathbf{V}_1 - j\omega M \mathbf{I}_2)/j\omega L_1$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2 \mathbf{I}_2}{L_1} \Rightarrow M = \sqrt{L_1 L_2} \quad (k = 1)$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{\sqrt{L_1 L_2}\,\mathbf{V}_1}{L_1} - \frac{j\omega L_1 L_2 \mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\,\mathbf{V}_1 = n\mathbf{V}_1$$



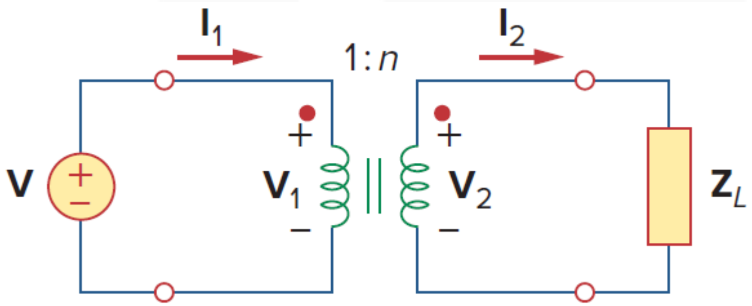
The circuit for deriving energy stored in a coupled circuit.

where $n = \sqrt{L_2/L_1}$ and is called the *turns ratio*. As $L_1, L_2, M \rightarrow \infty$ such that n remains the same, the coupled coils become an ideal transformer.

13.5 Ideal Transformer (3)

- A transformer will be ideal if it has the following properties:

1. Coils have very large reactances (L_1, L_2, $M \to \infty$).
2. Coupling coefficient is equal to unity ($k = 1$).
3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).



Relating primary and secondary quantities in an ideal transformer.

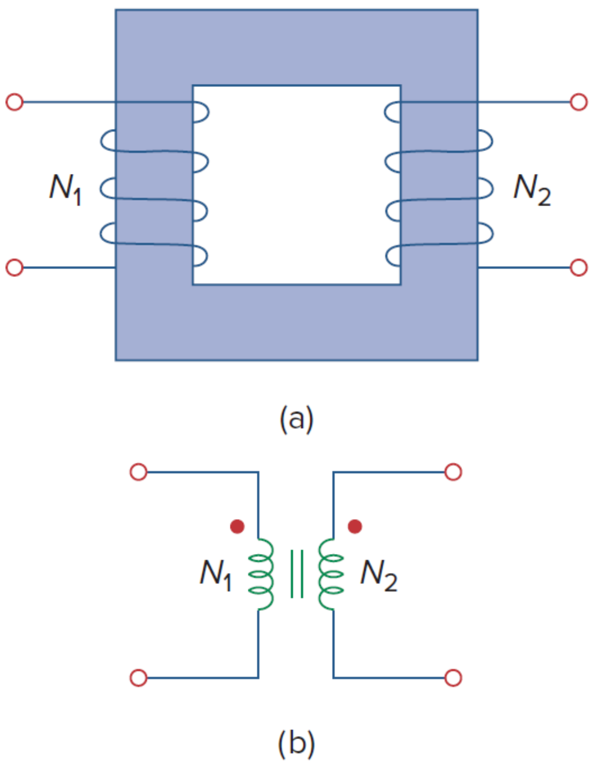
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

V2>V1→ step-up transformer

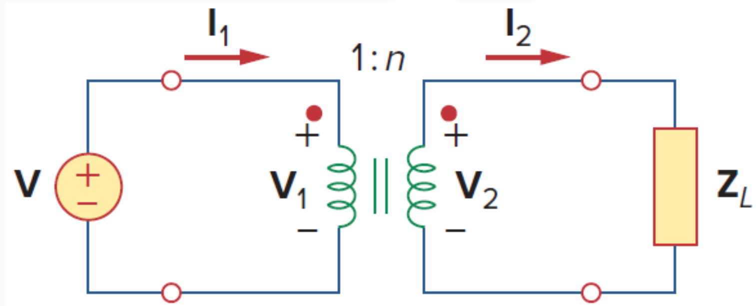
V2<V1→ step-down transformer

13.5 Ideal Transformer (3)

- An



(a) Ideal transformer, (b) circuit symbol for an ideal transformer.



Relating primary and secondary quantities in an ideal transformer.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

V2>V1→ step-up transformer

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13.5 Ideal Transformer (4)

Example 4

An ideal transformer is rated at 2400/120V, 9.6 kVA, and has 50 turns on the secondary side.

Calculate:

(a) the turns ratio,

(b) the number of turns on the primary side, and

(c) the current ratings for the primary and secondary windings.

Ans:

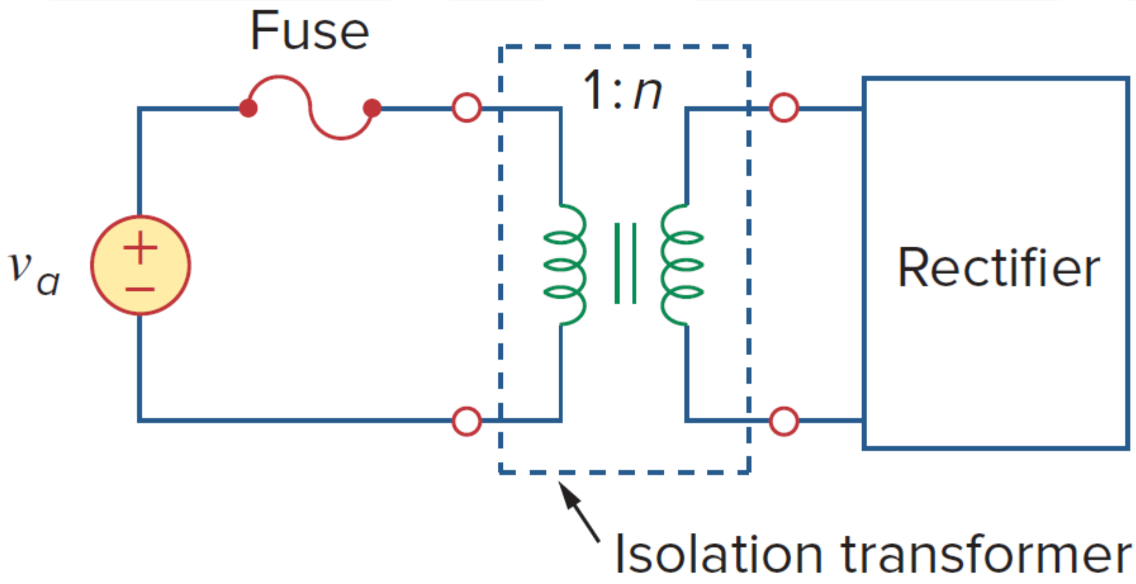
(a) This is a step-down transformer, n=0.05

(b) N1 = 1000 turns

(c) I1 = 4A and I2 = 80A

13.6 Applications (1)

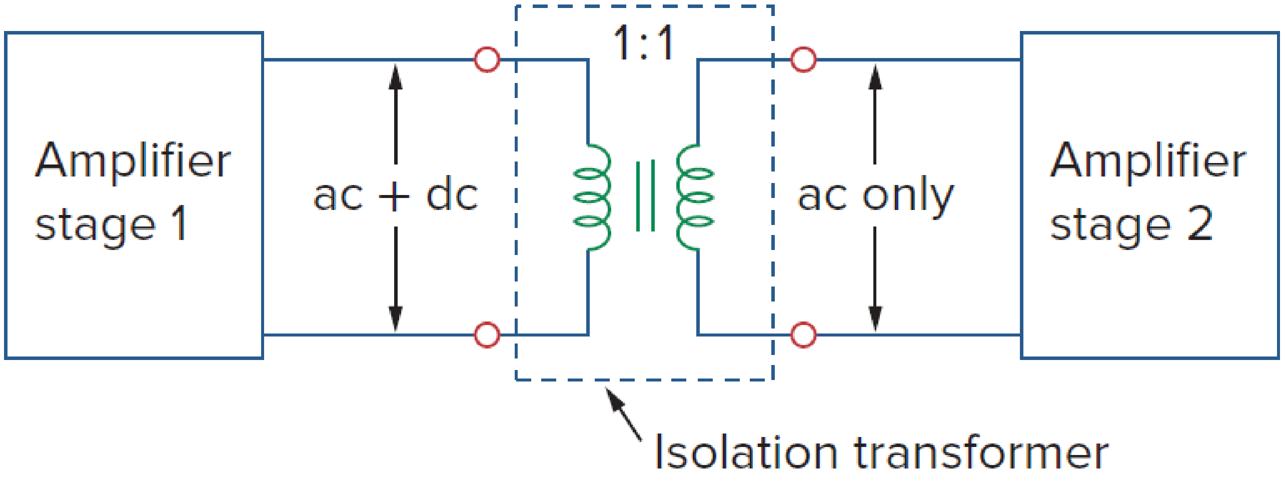
- Transformer as an Isolation Device to isolate ac supply from a rectifier



A transformer used to isolate an ac supply from a rectifier.

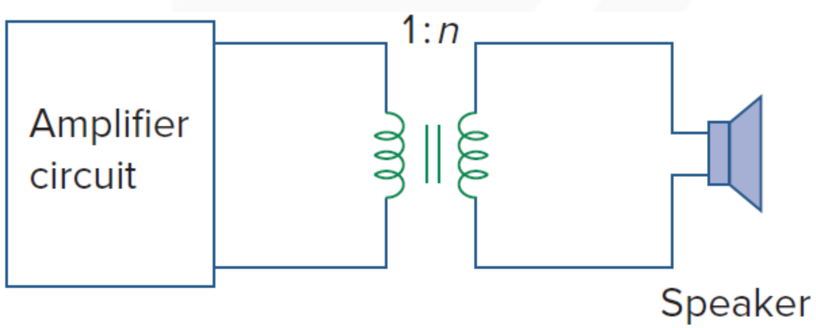
13.6 Applications (2)

- Transformer as an Isolation Device to isolate dc between two amplifier stages.

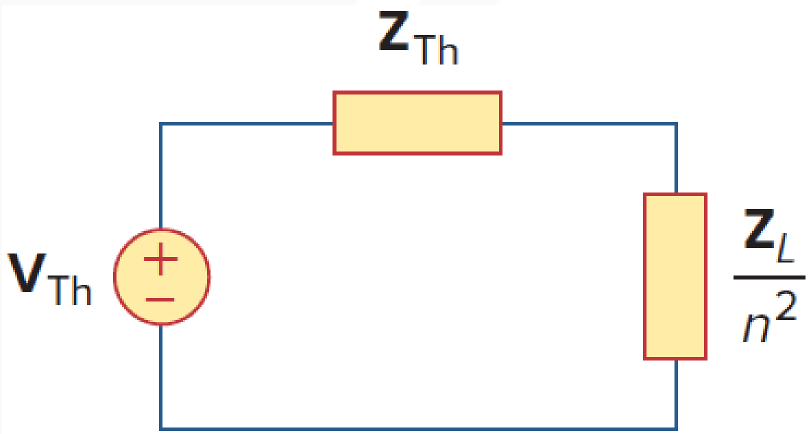


13.6 Applications (3)

- Transformer as a Matching Device



Using an ideal transformer to match the speaker to the amplifier



Equivalent circuit

13.6 Applications (4)

Ex.5 Calculate the turns ratio of an ideal transformer required to match a 100Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.2$; $V_L = 3V$

13.6 Applications (5)

- A typical power distribution system