Electrical Engineering 1
12026105

Chapter 11

AC Power Analysis

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Fully understand instantaneous and average power.
2. Understand the basics of maximum average power.
3. Understand effective or rms values and how to calculate them and to understand their importance.
4. Understand apparent power (complex power), power, and reactive power and power factor.
5. Understand power factor correction and the importance of its use.

วัตถุประสงค์การเรียนรู้

โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:

1. เข้าใจกำลังงานชั่วขณะและกำลังงานเฉลี่ย
2. เข้าใจพื้นฐานของกำลังงานเฉลี่ยสูงสุด
3. เข้าใจค่าประสิทธิผลหรือค่า rms และวิธีการคำนวณเพื่อเห็นถึงความสำคัญ
4. เข้าใจกำลังงานปรากฏ (กำลังงานเชิงซ้อน) กำลัง และปฏิกิริยากำลังและตัวประกอบกำลัง
5. เข้าใจ power factor correctionและการใช้งานมัน

AC Power Analysis Chapter 11

- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Conservation of AC Power
- Power Factor Correction
- Power Measurement

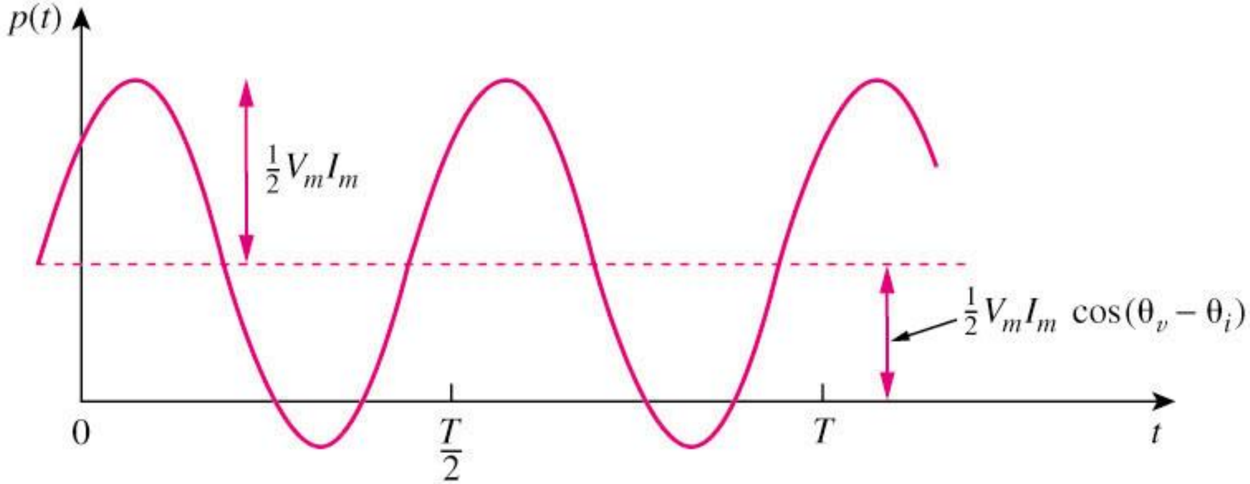
11.1 Instantaneous and Average Power

- The instantaneously power, $p(t)$

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power **Sinusoidal power at 2ωt**

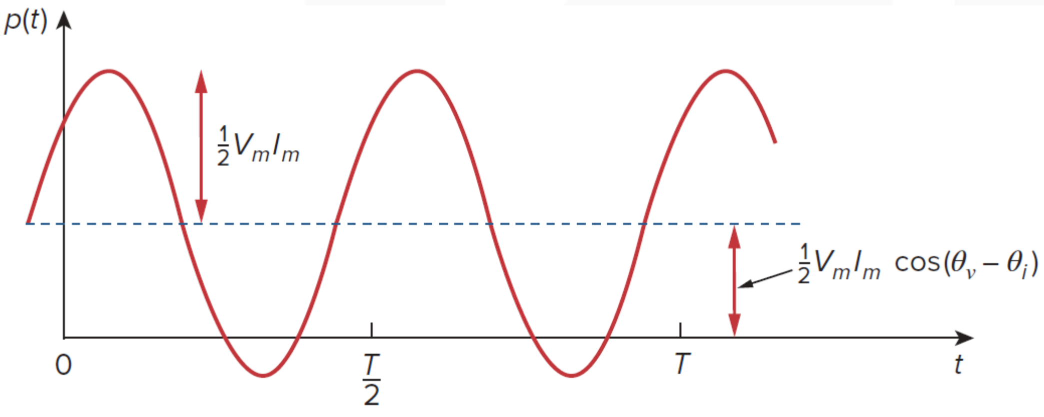


$p(t) > 0$: power is absorbed by the circuit;
$p(t) < 0$: power is absorbed by the source.

11.1 Instantaneous and Average Power

- The average power, P, is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m\, I_m \cos(\theta_v - \theta_i)$$



The instantaneous power $p(t)$ entering a circuit.

- $p(t)$ is time-varying while P does not depend on time.
- When $\theta_v = \theta_i$, the voltage and current are in phase. This implies a purely resistive circuit.
- When $\theta_v - \theta_i = \pm 90°$, This implies a purely reactive circuit.
- A purely reactive circuit absorbs no average power. $P = \frac{1}{2}V_m I_m \cos 90° = 0$
- A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

11.1 Instantaneous and Average Power

Example 1

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20^\circ)$$
$$i(t) = 15\sin(10t + 60^\circ)$$

$$i(t) = 15\sin(10t + 60^\circ) = 15\cos(10t + 60^\circ - 90^\circ) = 15\cos(10t - 30^\circ)$$

Answer: 385.7+600 cos(20t-10°)W, 385.7W

11.1 Instantaneous and Average Power

Example 2

A current I = 10∠30° flows through an impedance Z = 20∠ − 22°. Find the average power delivered to the impedance.

$$V = IZ = 200\angle 8°$$

$$P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) = \frac{200 \times 10}{2}\cos(8 - 30) = 927.2\text{W}$$

Answer: 927.2 W

11.2 Maximum Average Power Transfer

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\text{Th}}}{(R_{\text{Th}} + jX_{\text{Th}}) + (R_L + jX_L)} \quad \textbf{(11.14)}$$

From Eq. (11.11), the average power delivered to the load is

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{\text{Th}}|^2 R_L/2}{(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2} \quad \textbf{(11.15)}$$

Our objective is to adjust the load parameters R_L and X_L so that P is maximum. To do this we set $\partial P/\partial R_L$ and $\partial P/\partial X_L$ equal to zero. From Eq. (11.15), we obtain

$$\frac{\partial P}{\partial X_L} = -\frac{|\mathbf{V}_{\text{Th}}|^2 R_L (X_{\text{Th}} + X_L)}{[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2} \quad \textbf{(11.16a)}$$

$$\frac{\partial P}{\partial R_L} = \frac{|\mathbf{V}_{\text{Th}}|^2 [(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2 - 2R_L(R_{\text{Th}} + R_L)]}{2[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2} \quad \textbf{(11.16b)}$$

Setting $\partial P/\partial X_L$ to zero gives

$$X_L = -X_{\text{Th}} \quad \textbf{(11.17)}$$

and setting $\partial P/\partial R_L$ to zero results in

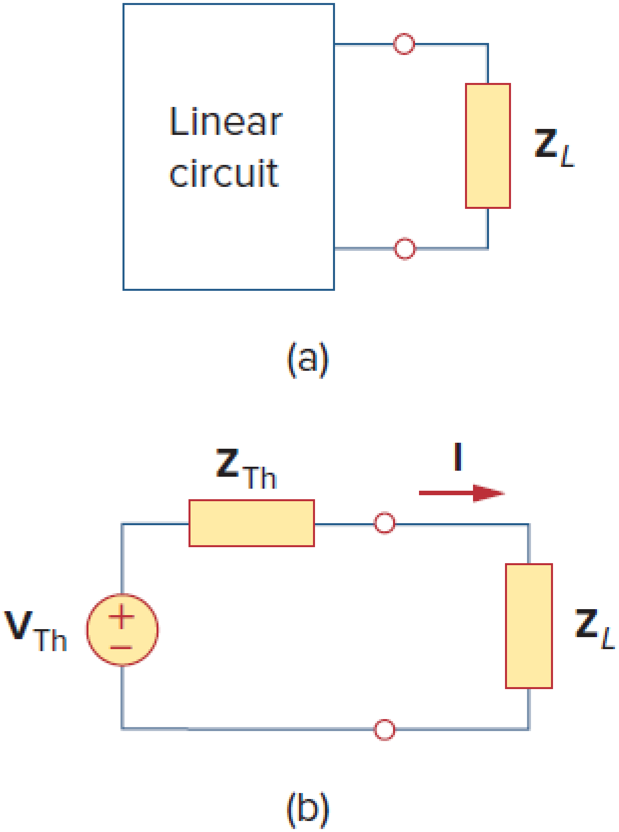
$$R_L = \sqrt{R_{\text{Th}}^2 + (X_{\text{Th}} + X_L)^2} \quad \textbf{(11.18)}$$

Combining Eqs. (11.17) and (11.18) leads to the conclusion that for maximum average power transfer, $\mathbf{Z}_L$ must be selected so that $X_L = -X_{\text{Th}}$ and $R_L = R_{\text{Th}}$, i.e.,

$$\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^* \quad \textbf{(11.19)}$$

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.

11.2 Maximum Average Power Transfer



Finding the maximum average power transfer: (a) circuit with a load, (b) the Thevenin equivalent.

$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

The maximum average power can be transferred to the load if

$$X_L = -X_{TH} \quad \text{and} \quad R_L = R_{TH}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

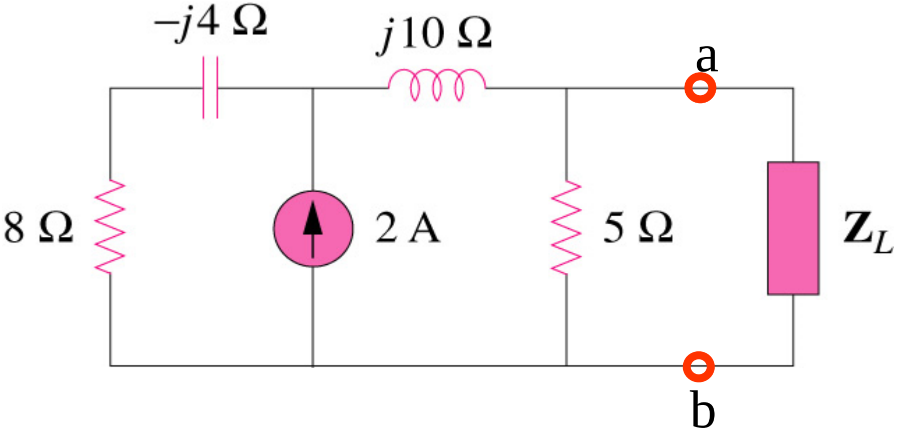
11.2 Maximum Average Power Transfer

Example 3

For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.

$$V_{TH} = \frac{8 - J4}{8 - J4 + 5 + J10} \times 5 = \frac{200(4 - J5)}{205} = 0.9756(4 - J5)$$

$$Z_{TH} = 5 \parallel 8 + J6 = \frac{5(8 + J6)}{13 + J6} = \frac{700 + J150}{205} = 3.415 + J0.7317$$



$$Z_L = \overline{Z_{TH}} = 3.415 - J0.7317$$

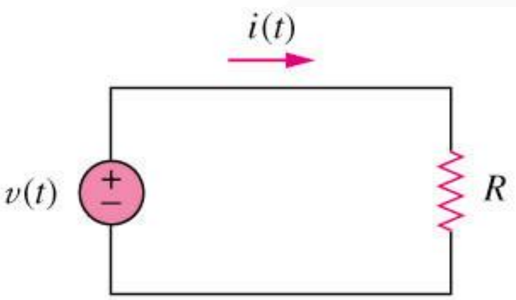
$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

$$P_{max} = \frac{|0.9756(4 - J5)|^2}{8 \times 3.415} = 1.429$$

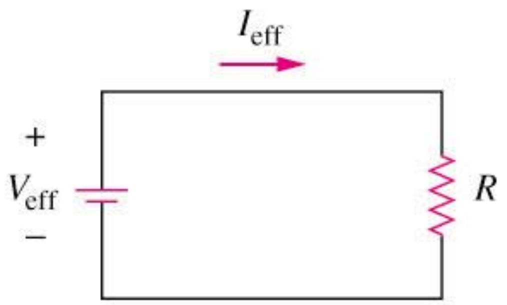
<u>Answer</u>: 3.415 – j0.7317Ω, 1.429W

11.3 Effective or RMS Value

The idea of effective value arises from the need to measure the effectiveness of a voltage or current source in delivering power to a resistive load.



(a)



(b)

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

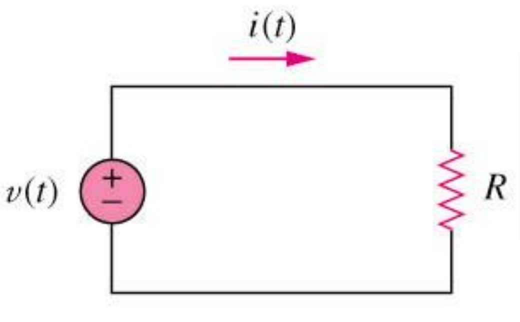
Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}$$

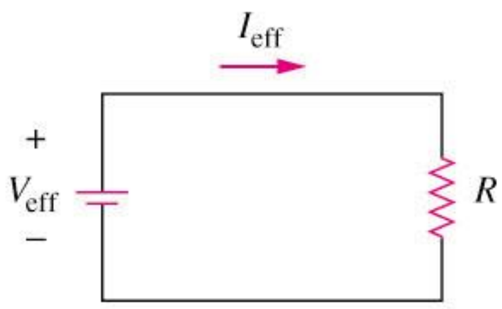
The effective value of a periodic signal is its root mean square (rms) value.

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.3 Effective or RMS Value



(a)



(b)

The rms value of a sinusoid $i(t) = I_m\cos(\omega t)$ is given by:

$$I_{rms}^2 = \frac{I_m}{\sqrt{2}}$$

The average power can be written in terms of the rms values:

$$I_{eff} = \frac{1}{2}V_m\,I_m\cos(\theta_v - \theta_i) = V_{rms}\,I_{rms}\cos(\theta_v - \theta_i)$$

Note: If you express amplitude of a phasor source(s) in rms, then all the answer as a result of this phasor source(s) must also be in rms value.

11.4 Apparent Power and Power Factor

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

Apparent Power, S   Power Factor, pf

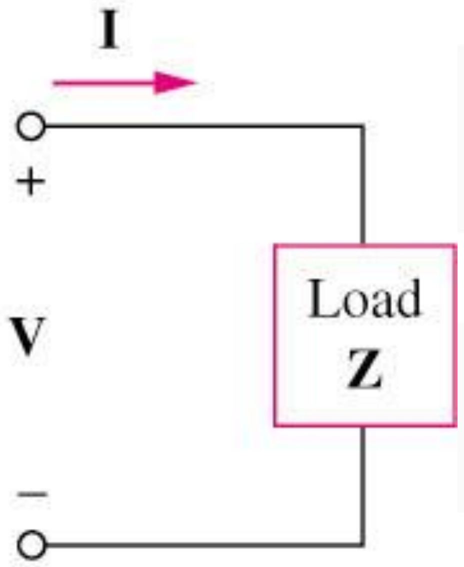
- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

11.4 Apparent Power and Power Factor

Purely resistive load (R)	$\theta_v - \theta_i = 0$, Pf = 1	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.5 Complex Power

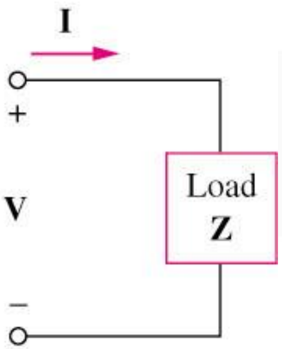
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} V\, I^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

11.5 Complex Power



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i = V_{rms} I_{rms} e^{j(\theta v - \theta i)}$$

$$\Rightarrow S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

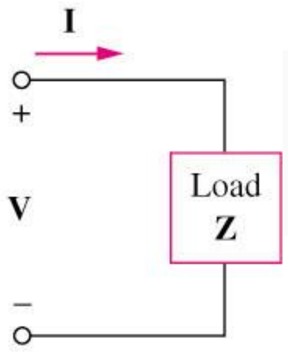
$$S = P + jQ$$

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

11.5 Complex Power



$$\Rightarrow S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$S = P + jQ$$

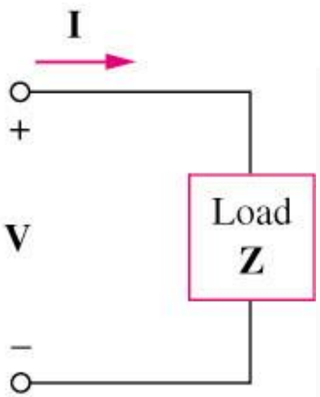
Apparent Power, $S = |\mathbf{S}| = V_{rms} * I_{rms} = \sqrt{P^2 + Q^2}$

Real power, $P = \mathrm{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$

Reactive Power, $Q = \mathrm{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$

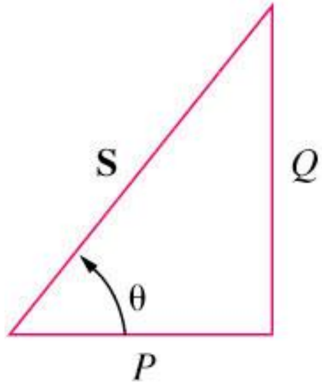
Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

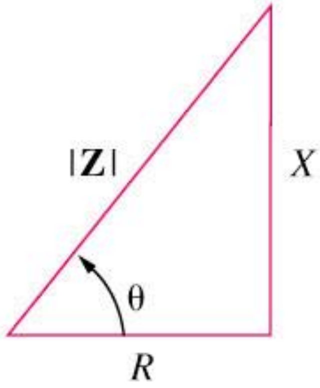
11.5 Complex Power

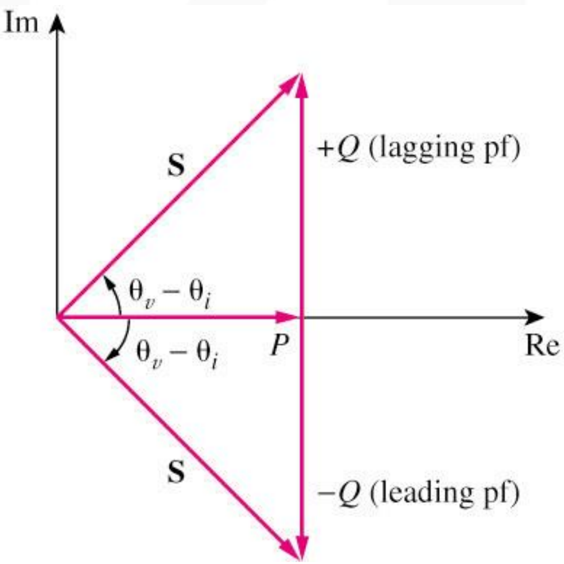


$$\Rightarrow S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$S = P + jQ$$



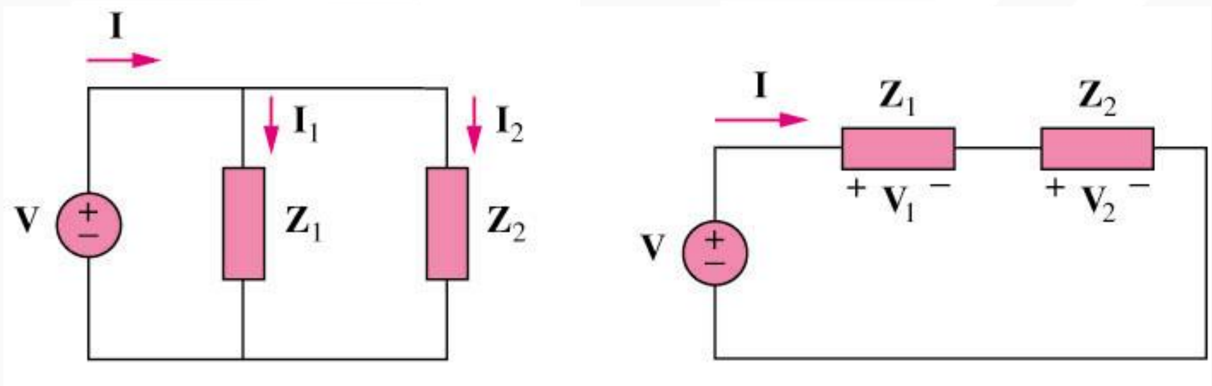




Power Triangle | Impedance Triangle | Power Factor

11.6 Conservation of AC Power

The complex real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.



(a) (b)

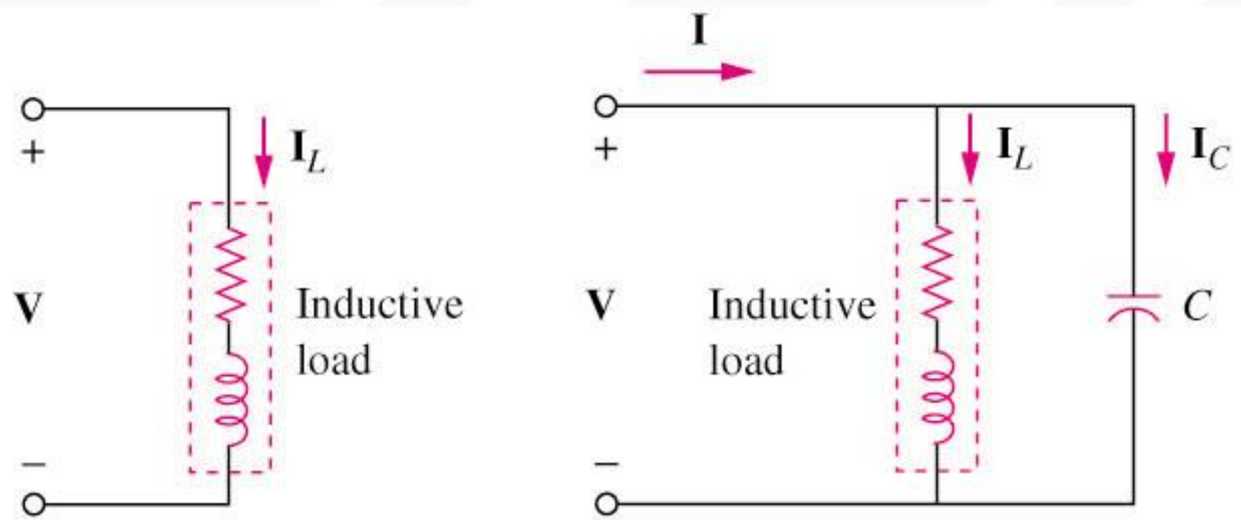
For parallel connection:

$$\overline{S} = \frac{1}{2}\overline{V}\,\overline{I}^{*} = \frac{1}{2}\overline{V}\,(\overline{I_1}^{*} + \overline{I_2}^{*}) = \frac{1}{2}\overline{V}\,\overline{I_1}^{*} + \frac{1}{2}\overline{V}\,\overline{I_2}^{*} = \overline{S_1} + \overline{S_2}$$

The same results can be obtained for a series connection.

11.7 Power Factor Correction

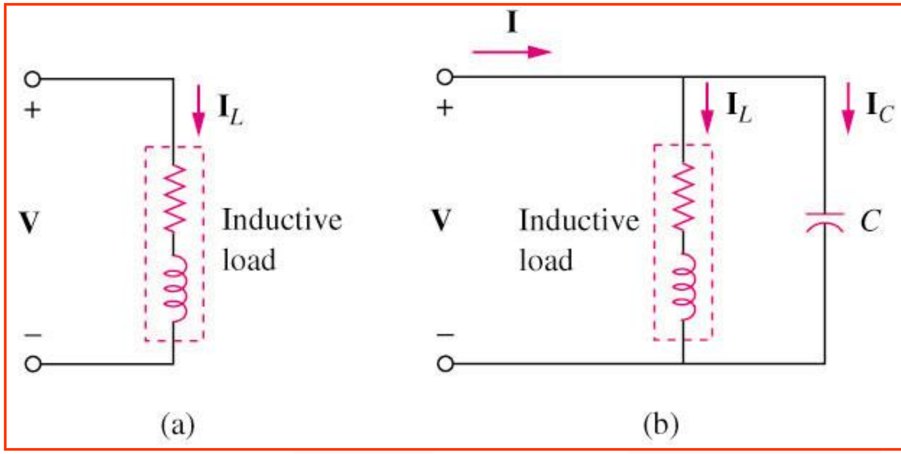
Power factor correction is the process of increasing the power factor without altering the voltage or current to the original load.

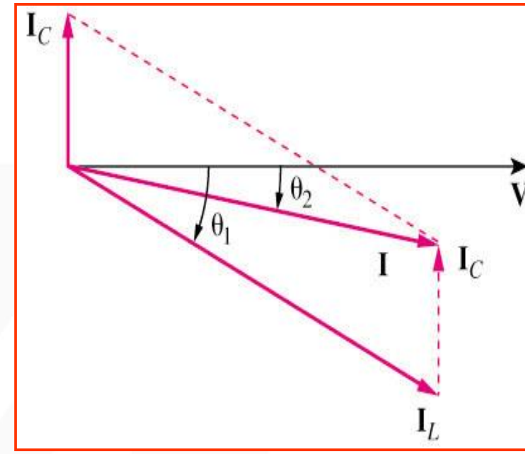


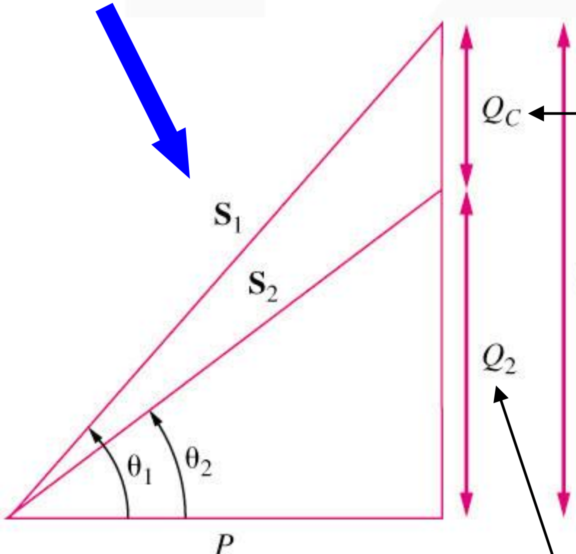
(a) (b)

Power factor correction is necessary for economic reason.

11.7 Power Factor Correction







$Q_c = Q_1 – Q_2$

$= P (\tan \theta_1 - \tan \theta_2)$

$= \omega C V_{2rms}$

$Q_1 = S_1 \sin \theta_1$

$= P \tan \theta_1$

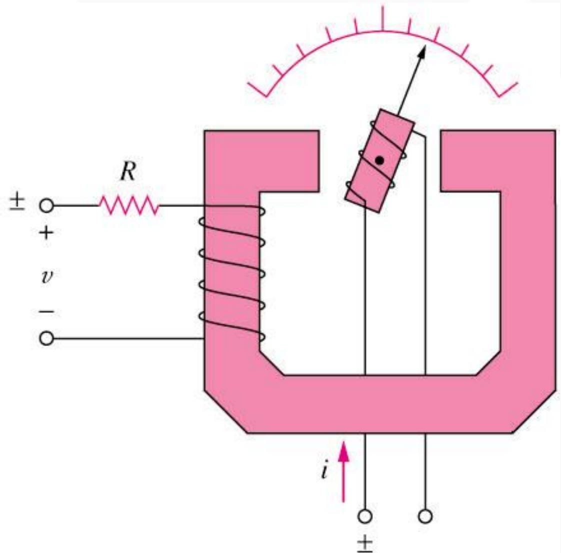
$Q_2 = P \tan \theta_2$

$P = S_1 \cos \theta_1$

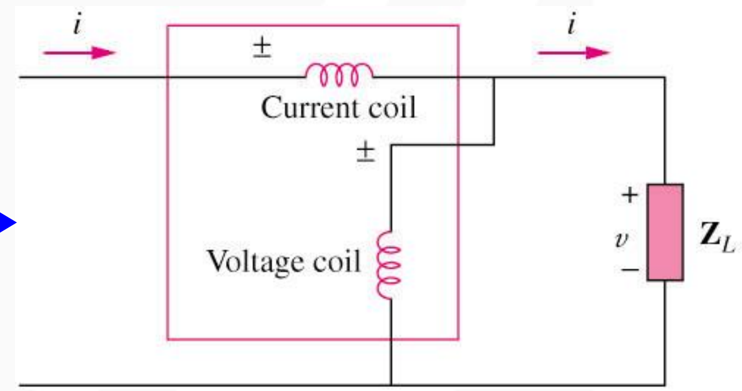
$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

11.8 Power Measurement

The <u>wattmeter</u> is the instrument for measuring the average power.



The basic structure



Equivalent Circuit with load

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$