

Electrical Engineering 1

12026105

Lecture 6

Capacitors and Inductors

Capacitors and Inductors

6.1 Capacitors (ตัวเก็บประจุ)

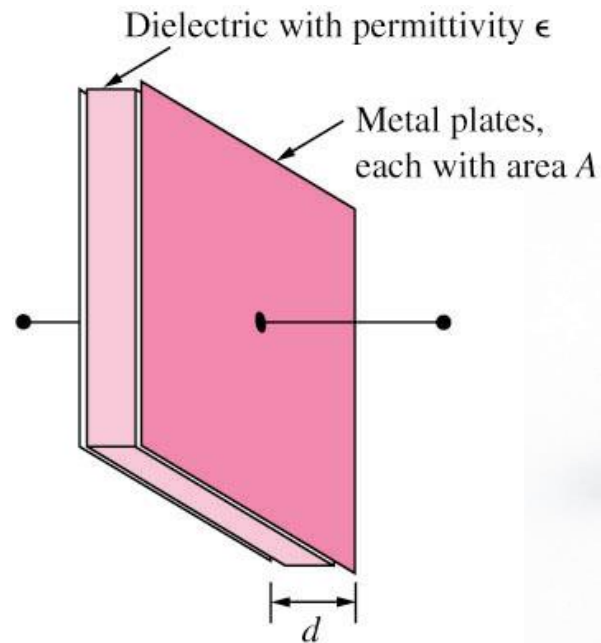
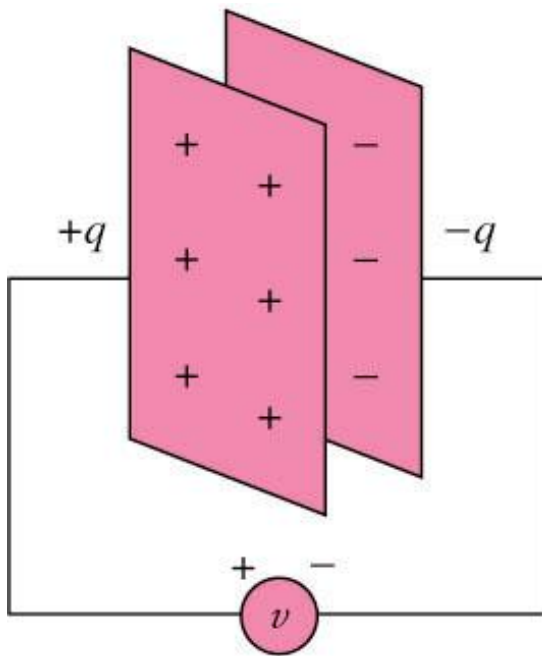
6.2 Series and Parallel Capacitors (ต่ออนุกรม,ขนานตัวเก็บประจุ)

6.3 Inductors (ขดลวดเหนี่ยวนำ)

6.4 Series and Parallel Inductors (ต่ออนุกรม,ขนานขดลวดเหนี่ยวนำ)

6.1 Capacitors (1)

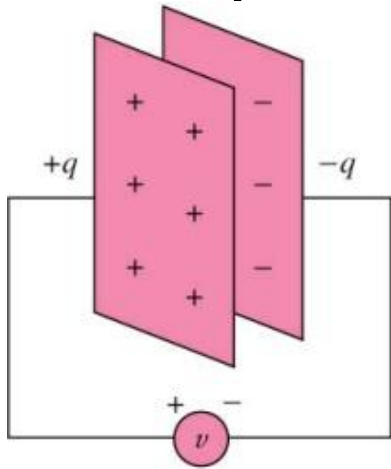
- A capacitor is a passive element designed to **store energy** in its **electric field**.



- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors (2)

- **Capacitance C** is the ratio of the charge q on one plate of a capacitor to the voltage difference V between the two plates, measured in farads (F).



$$q = C v$$

and

$$C = \frac{\epsilon A}{d}$$

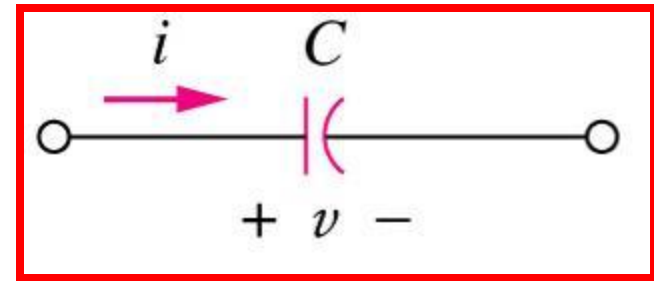
- Where ϵ is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

6.1 Capacitors (3)

- If i is flowing into the +ve terminal of C

– Charging $\rightarrow I$ is +ve

– Discharging $\rightarrow i$ is -ve



- The current-voltage relationship of capacitor according to above convention is

$$i = C \frac{dv}{dt}$$

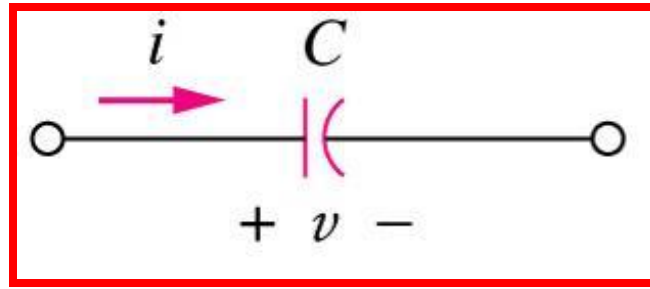
and

$$v = \frac{1}{C} \int_{t_0}^t i dt + v(t_0)$$

6.1 Capacitors (4)

- The energy, w , stored in the capacitor is

$$w = \frac{1}{2} C v^2$$



- A capacitor is

- an **open circuit** to dc (กระแสตรง) ($\frac{dv}{dt} = 0, i = C \frac{dv}{dt}$)
- its voltage **cannot change abruptly.**

6.1 Capacitors (5)

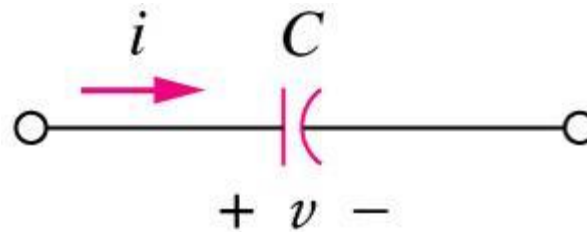
Example 1

The current through a 100- μF capacitor is

$$i(t) = 50\sin(120\pi t) \text{ mA.}$$

Calculate the voltage across it at $t = 1 \text{ ms}$ and $t = 5 \text{ ms}$.

Take $v(0) = 0$.



$$v = \frac{1}{C} \int_{t_0}^t i \, dt + v(t_0)$$

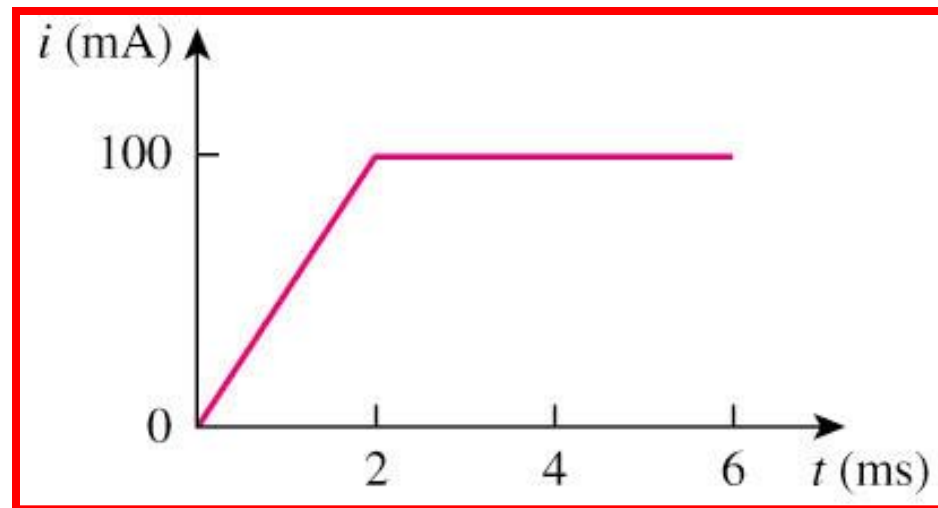
Answer: $v(1\text{ms}) = 93.14\text{mV}$, $v(5\text{ms}) = 1.7361\text{V}$

6.1 Capacitors (6)

Example 2

An initially uncharged 1-mF capacitor has the current shown below across it.

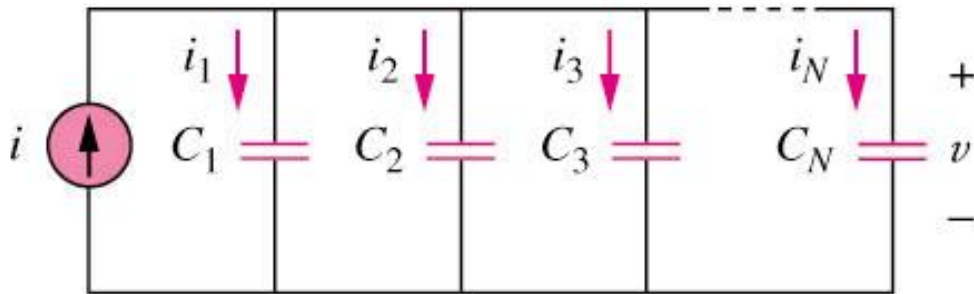
Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.



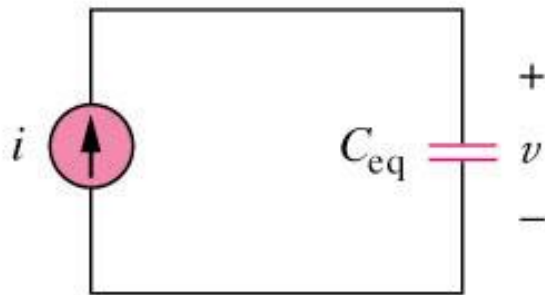
Answer: $v(2\text{ms}) = 100$ mV, $v(5\text{ms}) = 500$ mV

6.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of N **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

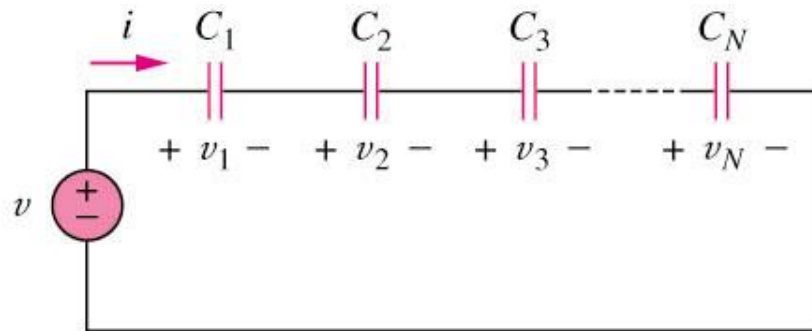


(b)

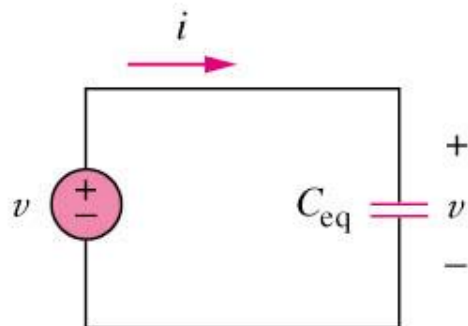
$$C_{eq} = C_1 + C_2 + \dots + C_N$$

6.2 Series and Parallel Capacitors (2)

- The equivalent capacitance of N **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)



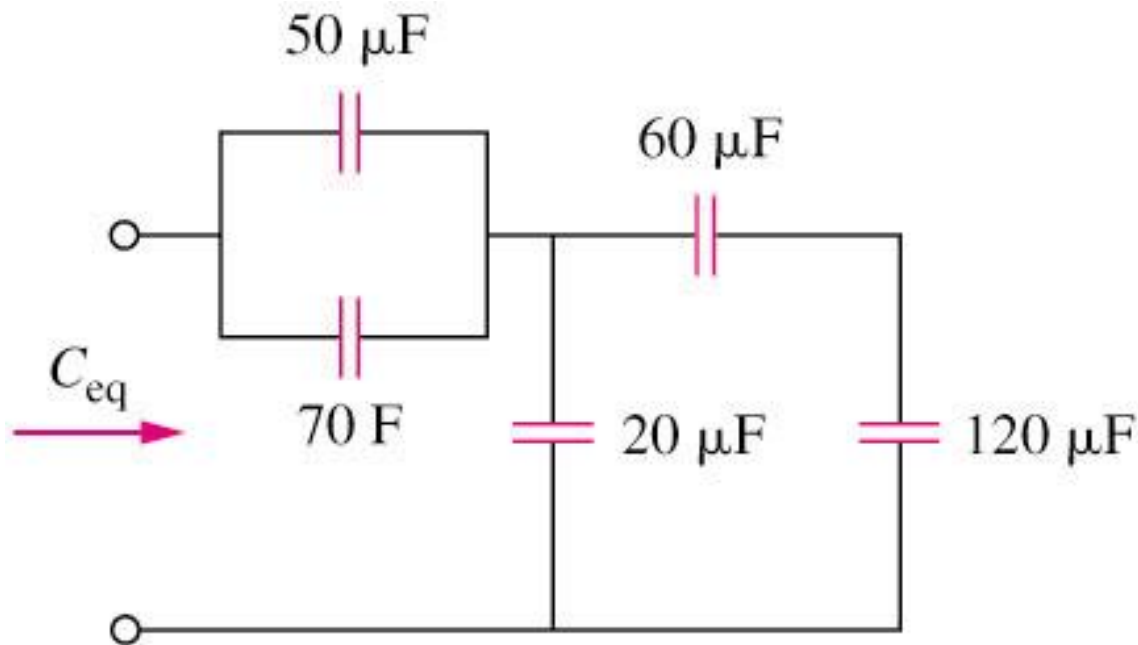
(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

6.2 Series and Parallel Capacitors (3)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:

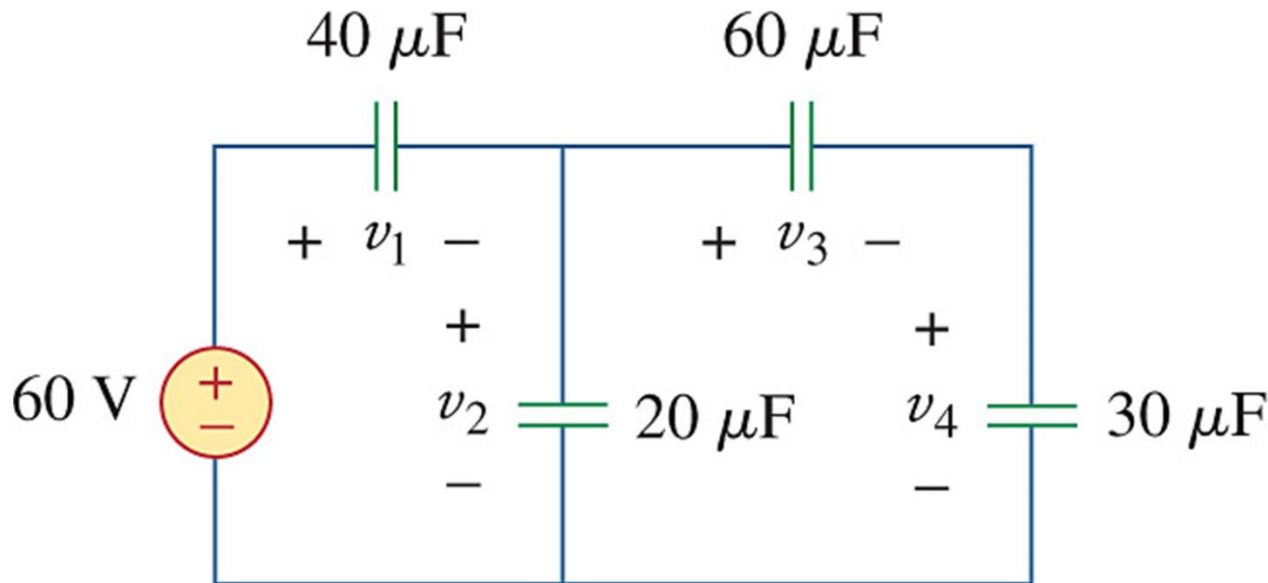


Answer: $C_{eq} = 40 \mu\text{F}$

6.2 Series and Parallel Capacitors (4)

Example 4

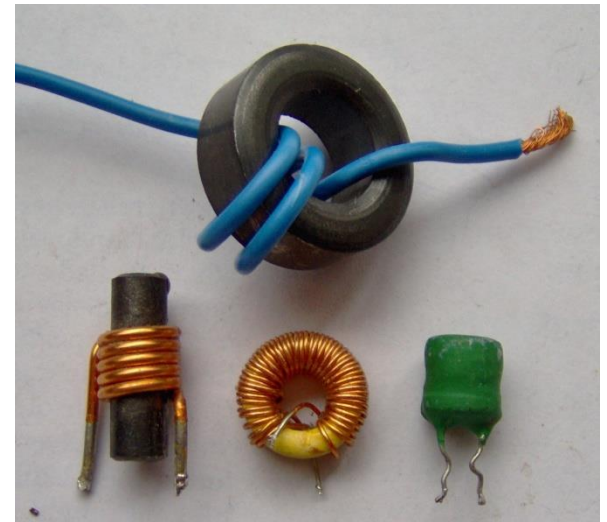
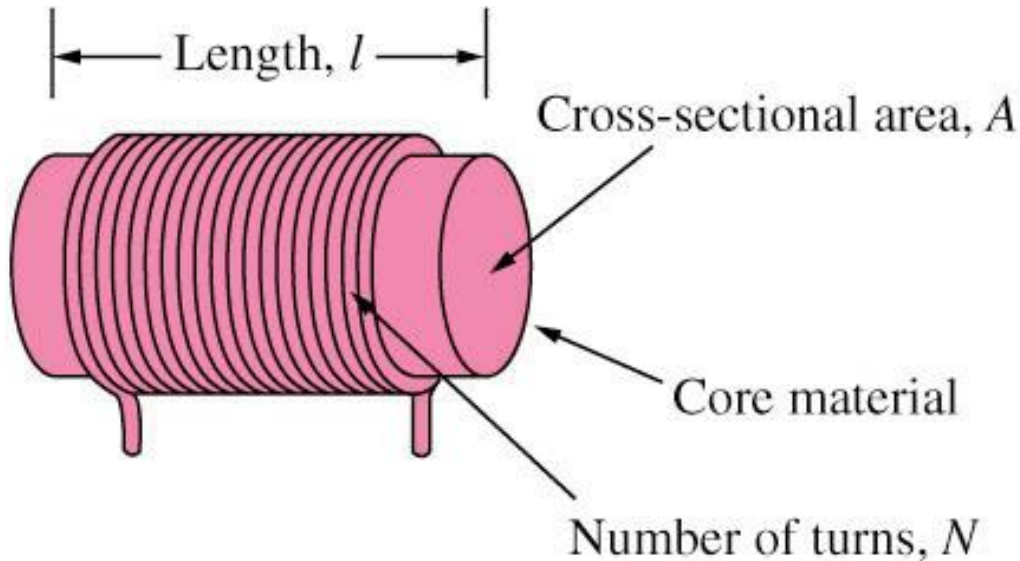
Find the voltage across each of the capacitors in the circuit shown below:



Answer: $v_1 = 30\text{V}$, $v_2 = 30\text{V}$, $v_3 = 10\text{V}$, $v_4 = 20\text{V}$

6.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

6.3 Inductors (2)

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

$$v = L \frac{d i}{d t}$$

and

$$L = \frac{\mu A N^2}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6})

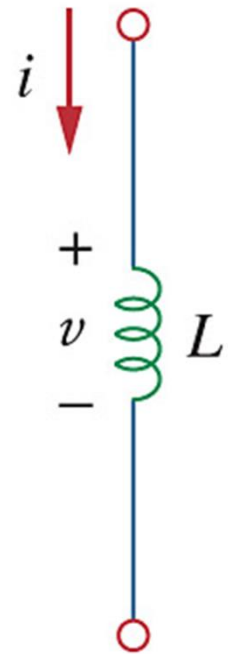
6.3 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

- The power stored by an inductor:

$$w = \frac{1}{2} L i^2$$



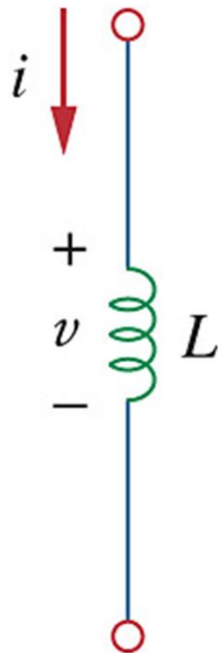
- An inductor acts like a short circuit to dc ($di/dt = 0$) and its current cannot change abruptly.

6.3 Inductors (4)

Example 5

The terminal voltage of a 2-H inductor is $v = 10(1-t)$ V

Find the current flowing through it at $t = 4$ s and the energy stored in it within $0 < t < 4$ s. Assume $i(0) = 2$ A.

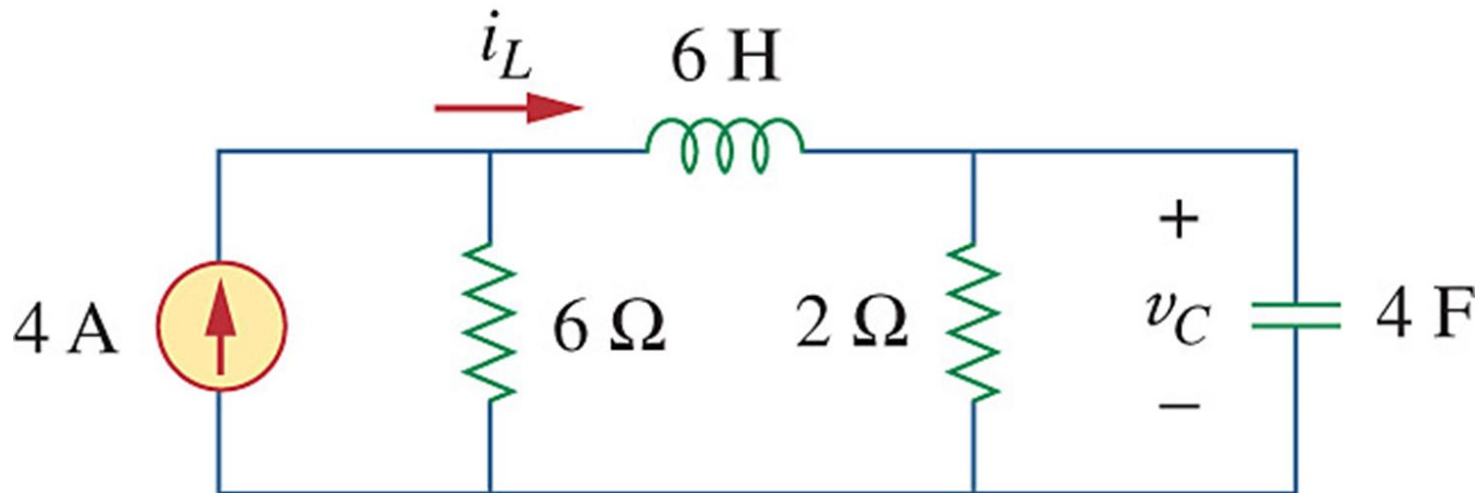


Answer: $i(4s) = -18$ A, $w(4s) = 320$ J

6.3 Inductors (5)

Example 6

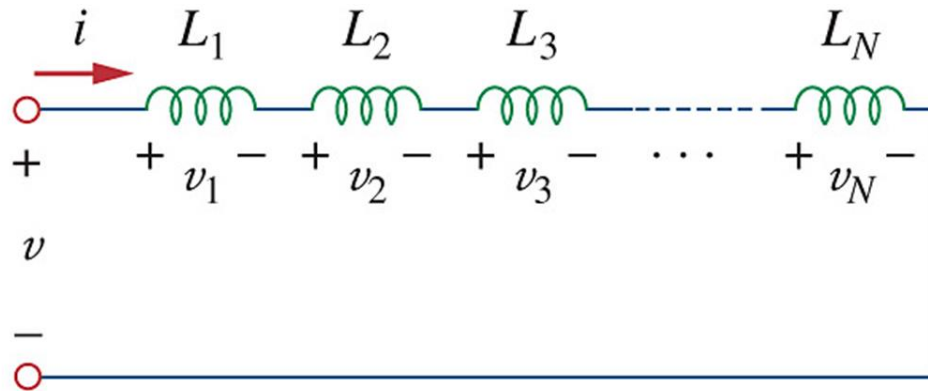
Determine v_C , i_L , and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



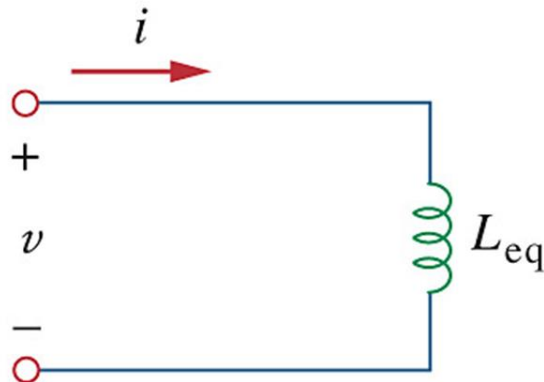
Answer: $v_C = 15\text{V}$, $i_L = 7.5\text{A}$, $w_C = 450\text{J}$, $w_L = 168.75\text{J}$

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)

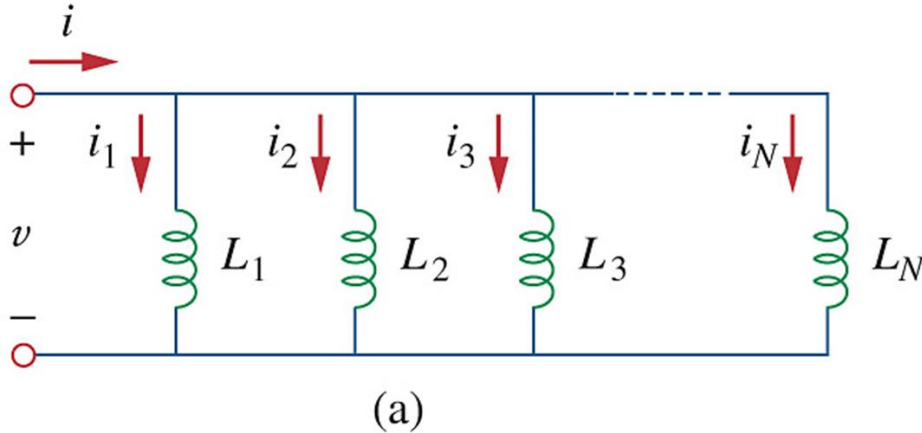


(b)

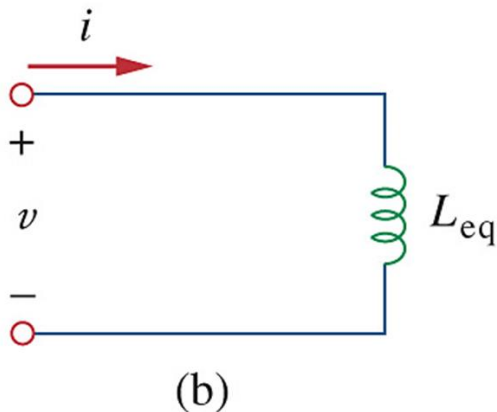
$$L_{eq} = L_1 + L_2 + \dots + L_N$$

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



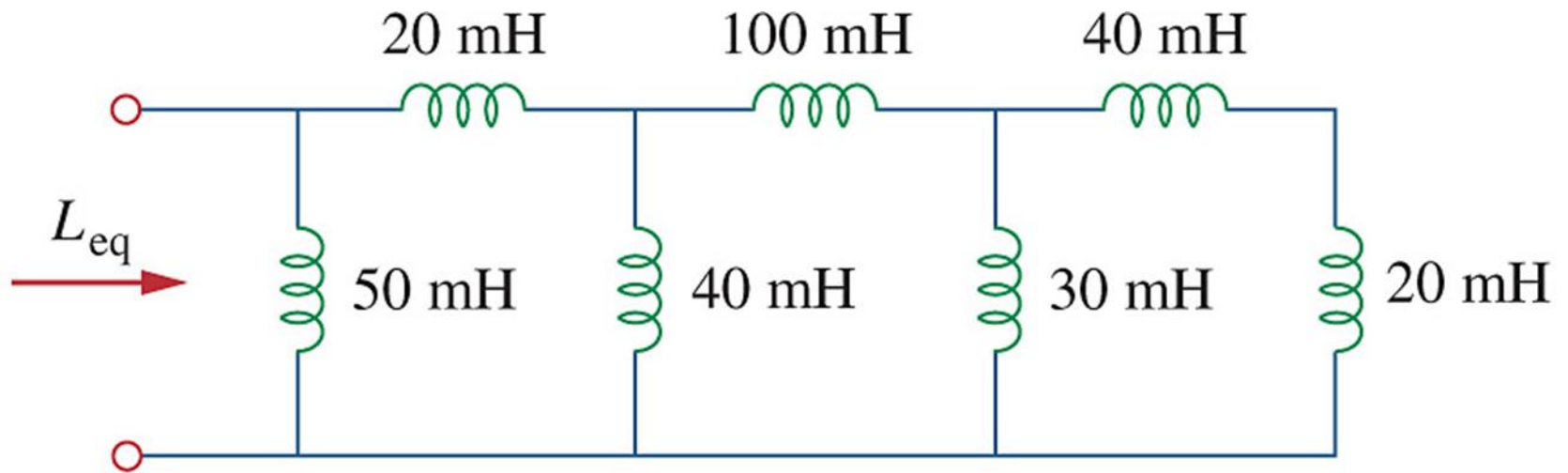
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$



6.4 Series and Parallel Capacitors (3)

Example 7




Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



Answer: $L_{eq} = \underline{25\text{mH}}$

6.4 Series and Parallel Capacitors (4)

- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 <p>Resistance</p>	ohms (Ω)	$v = Ri$ <p>(Ohm's law)</p>	$i = \frac{v}{R}$	$p = vi = i^2 R$
 <p>Inductance</p>	henries (H)	$v = L \frac{di}{dt}$	$i = \frac{1}{L} \int v dt + k_1$	$p = vi = Li \frac{di}{dt}$
 <p>Capacitance</p>	farads (F)	$v = \frac{1}{C} \int i dt + k_2$	$i = C \frac{dv}{dt}$	$p = vi = Cv \frac{dv}{dt}$