Electrical Engineering 1
12026105

Lecture 10
Sinusoidal Steady-State Analysis

Sinusoidal Steady-State Analysis

10.1 Basic Approach

10.2 Nodal Analysis

10.3 Mesh Analysis

10.4 Superposition Theorem

10.5 Source Transformation

10.6 Thevenin and Norton Equivalent Circuits

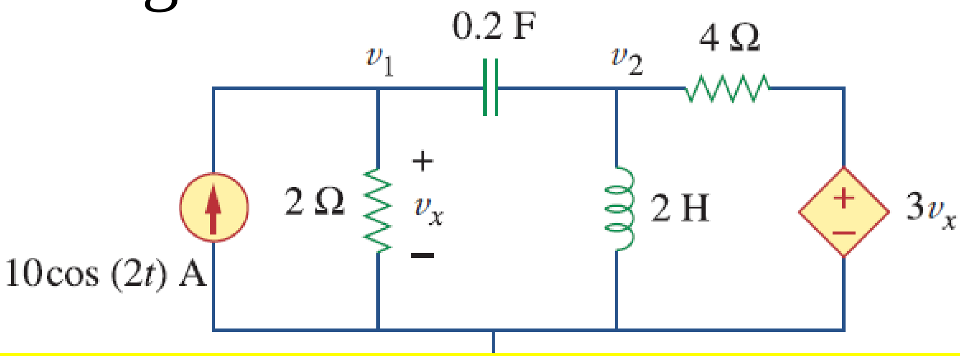
10.1 Basic Approach (1)

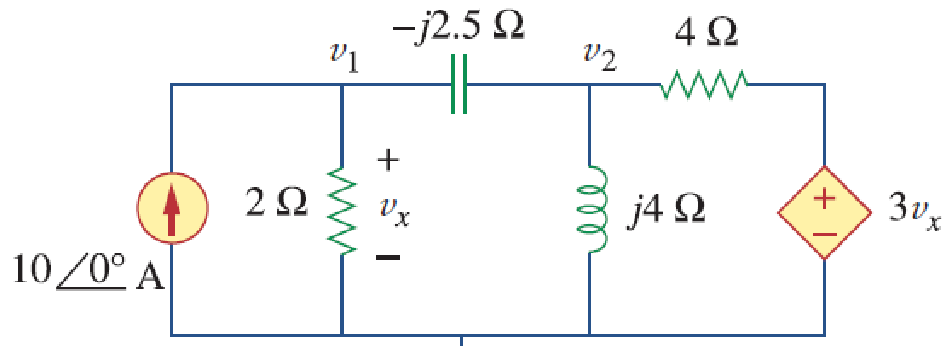
Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

10.2 Nodal Analysis (1)

Example 1 Using nodal analysis, find v_1 and v_2 in the circuit of figure below.





$$\frac{v_1 - v_2}{-j2.5} + \frac{v_1}{2} = 10\angle 0^\circ \Leftrightarrow (-1 + j1.25)v_1 + v_2 = 25\angle 90^\circ$$

$$\frac{v_2 - v_1}{-j2.5} + \frac{v_2}{j4} + \frac{v_2 - 3v_1}{4} = 0 \Leftrightarrow -4(v_2 - v_1) + 2.5v_2 + j2.5v_2 - j7$$

$$(4 - j7.5)v_1 = (1.5 - j2.5)v_2 \Leftrightarrow v_1 = \left(\frac{24.75 + j1.25}{72.25}\right)v_2$$

$$(-1 + j1.25)\left(\frac{24.75 + j1.25}{72.25}\right)v_2 + v_2 = 25\angle 90^\circ \Leftrightarrow (0.636 + j$$

$$(0.757\angle 32.8^\circ)v_2 = 25\angle 90^\circ$$

$$v_2 = \frac{25\angle 90^\circ}{0.757\angle 32.8^\circ} = 33.02\angle 57.2^\circ$$

$$v_1 = \left(\frac{24.75 + j1.25}{72.25}\right)33.02\angle 57.2^\circ$$

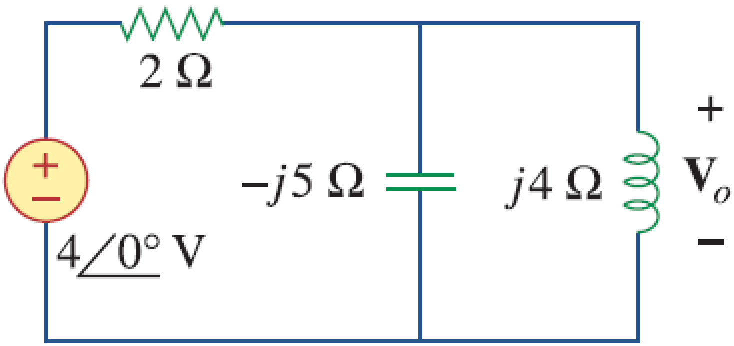
$$= (0.343\angle 2.89^\circ)(33.02\angle 57.2^\circ)$$

$$= 11.32\angle 60.09^\circ$$

Answer: $v_1(t)$ = **11.32 cos(2t + 60.01°) V** **$v_2(t)$ = 33.02 cos(2t + 57.12°) V**

10.3 Mesh Analysis (1)

<u>Example 2</u> Find V_0 using node analysis.



$$\frac{V_0}{j4}+\frac{V_0}{-j5}+\frac{V_0-4}{2}=0$$

$$V_0\left(\frac{1}{j4}+\frac{1}{-j5}+\frac{1}{2}\right)=2$$

$$V_0(-j0.25+j0.2+0.5)=2$$

$$V_0=\frac{2}{0.5-j0.05}=\frac{2}{0.5-j0.05}\times\frac{0.5+j0.05}{0.5+j0.05}=\frac{1+j0.1}{0.2525}=3.98\angle 5.71^{\circ}A$$

Answer: $V_0=3.98\angle 5.71^{\circ}A$

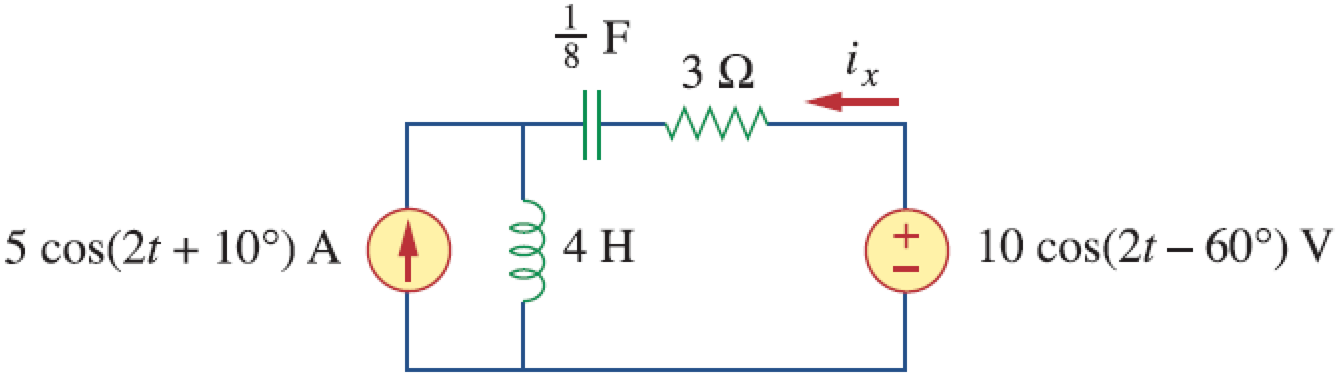
10.4 Superposition Theorem (1)

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.4 Superposition Theorem (2)

Example 3 Calculate i_x in the circuit of figure shown below using the superposition theorem.



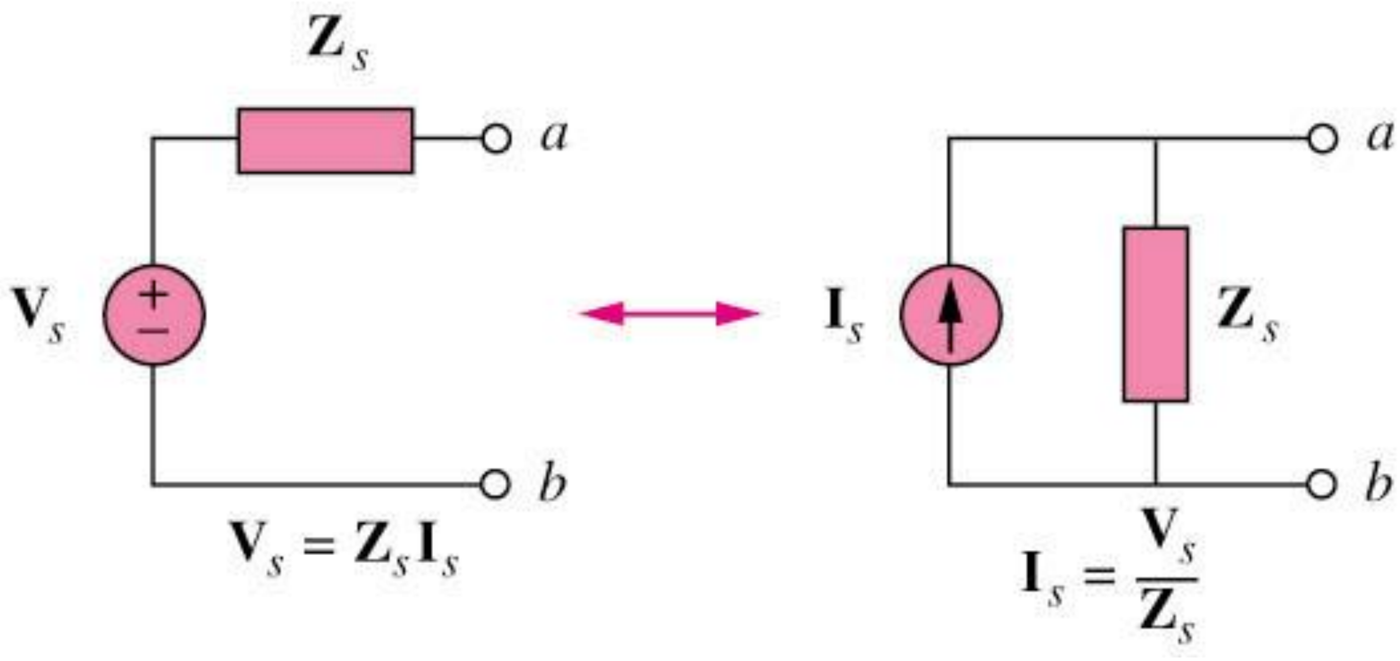
$$(i_x)_1 = \frac{10\cos(2t-60°)}{3+j8-j4} = \frac{(10\angle -60°)(5\angle -53.13°)}{25} = 2\angle -113.13° = -0.7856 - j1.839$$

$$(i_x)_2 = -\frac{j8}{3+j8-j4} 5\angle 10° = \frac{-32-j24}{25} 5\angle 10° = 1.6\angle 216.87° \times 5\angle 10° = 8\angle 226.87° = -5.47 - j5.84$$

$$i_x = (i_x)_1 + (i_x)_2 = -6.26 - j7.68 = 9.9\angle -129°$$

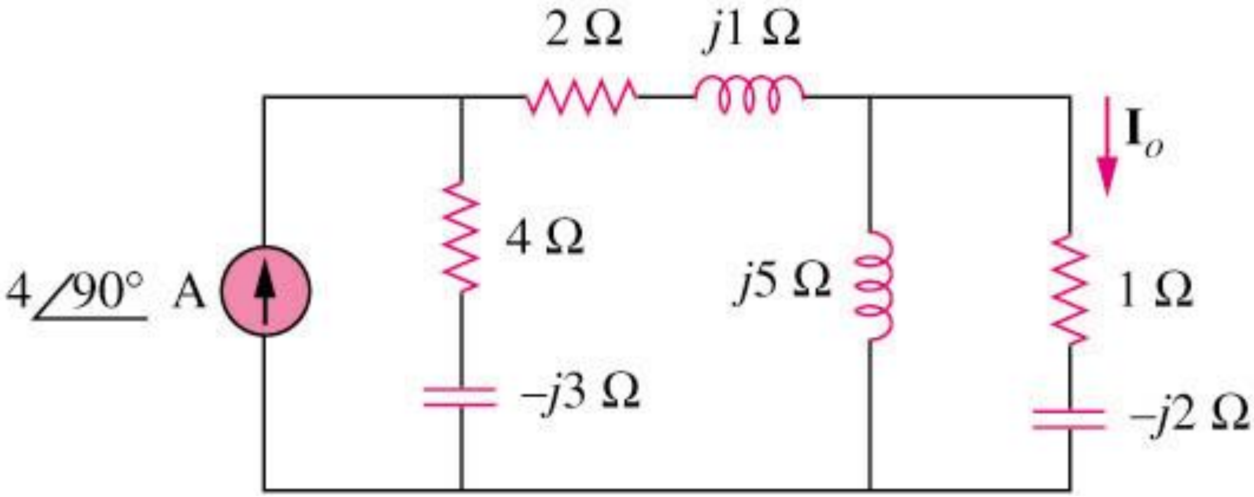
$\text{Answer}: i_x = 9.902\cos(2t - 129.17°)A$

10.5 Source Transformation (1)



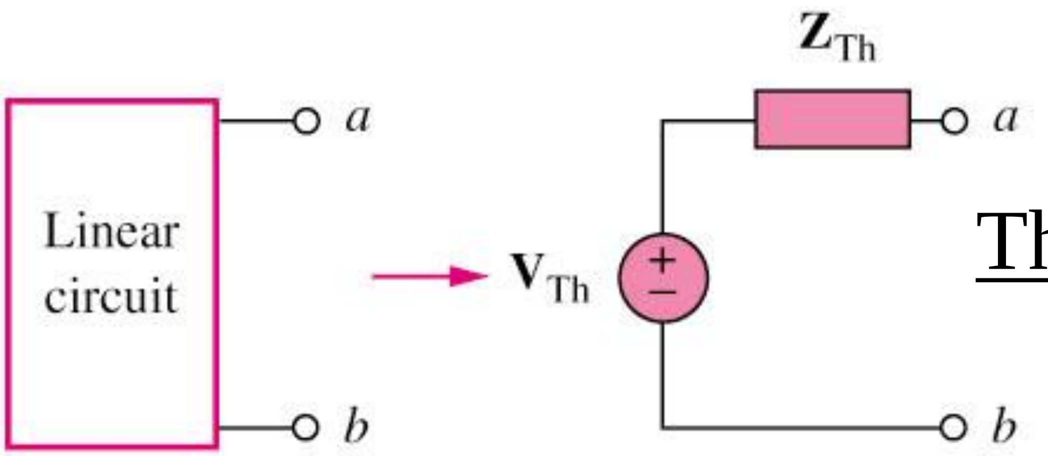
10.5 Source Transformation (2)

<u>Example 4</u> Find I_o in the circuit of figure below using the concept of source transformation.

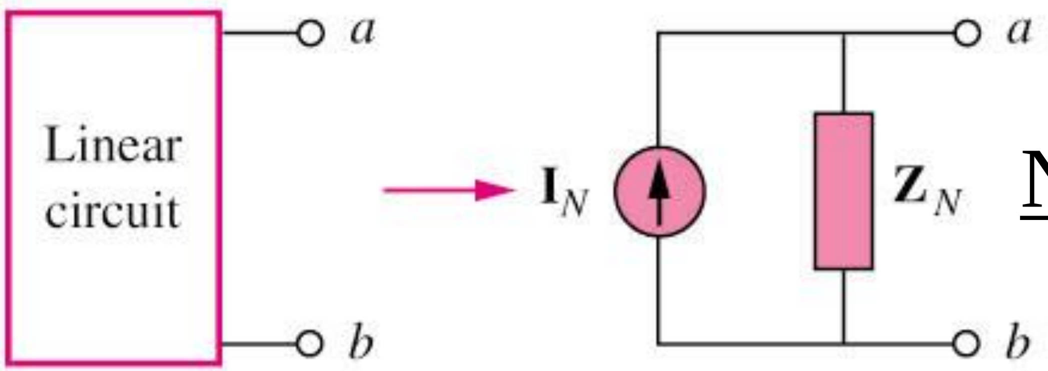


$I_0 = 3.288\angle 99.46^{\circ}\ A$

10.6 Thevenin&Norton Equiv Circuits (1)



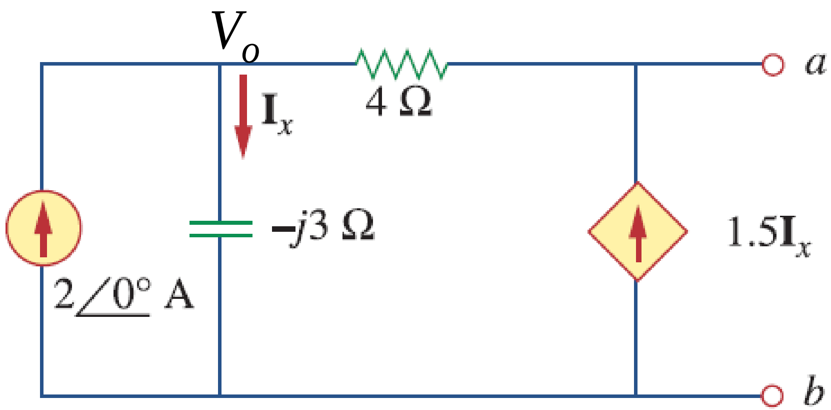
Thevenin transform

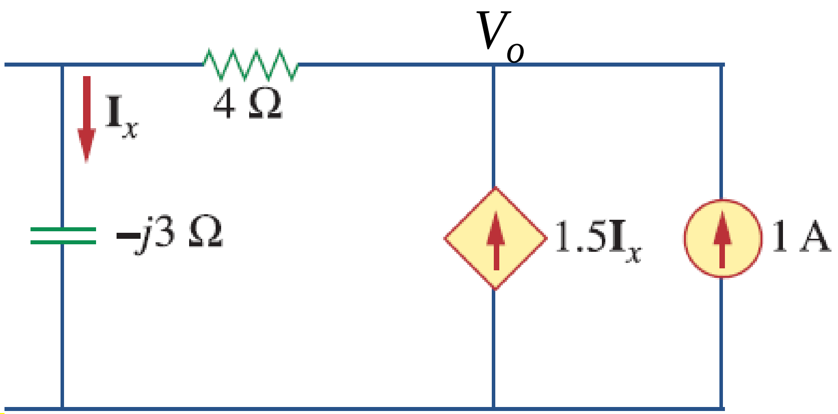


Norton transform

10.6 Thevenin&Norton Equiv Circuits (2)

Example 5 Find the Thevenin equivalent at terminals a–b of the circuit below.





$$2+1.5I_x = I_x \rightarrow I_x = -4$$

$$V_o = I_x(-j3) = j12$$

$$V_{TH} = V_o + 6I_x = -24 + j12$$

$$1+1.5I_x = I_x \rightarrow I_x = -2$$

$$V_o = I_x(4 - j3) = -8 + j6$$

$$Z_{TH} = \frac{V_o}{1} = -8 + j6$$

$$Z_{TH} = -8 + j6\Omega$$

$$V_{TH} = (-24 + j12)V$$