Electrical Engineering 1
12026105

Chapter 4

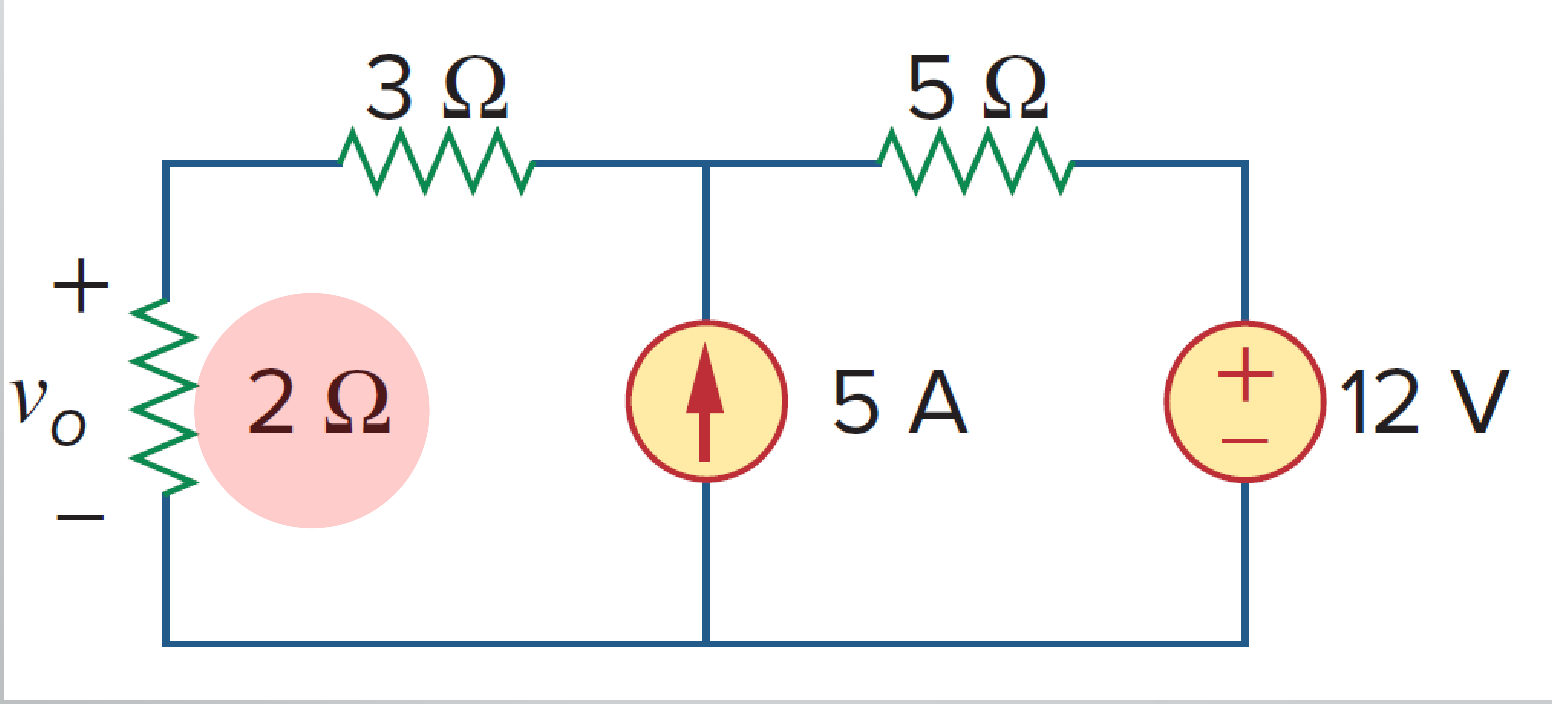
Circuit Theorems

Circuit Theorems - Chapter 4

- Introduction
- Linearity Property
- Superposition
- Source Transformation
- Thevenin’s Theorem
- Norton’s Theorem
- Maximum Power Transfer

4.1 Introduction

Are there any other alters to find the voltage across 2Ω ?



What are they? And how?

$v_o = 7.4$ V

Can you work it out by inspection?

4.2 Linearity Property (1)

- It is the property of an element describing *a linear relationship between cause and effect*.
- A linear circuit is one whose output is linearly related (directly proportional) to its input.
- The property is a combination of both *homogeneity (scaling)* and *additivity*.

Homogeneity (scaling) property:

(If i is increased by a constant k, then v increases correspondingly by k)

$$v = iR \rightarrow \quad kv = kiR$$

Additive property:

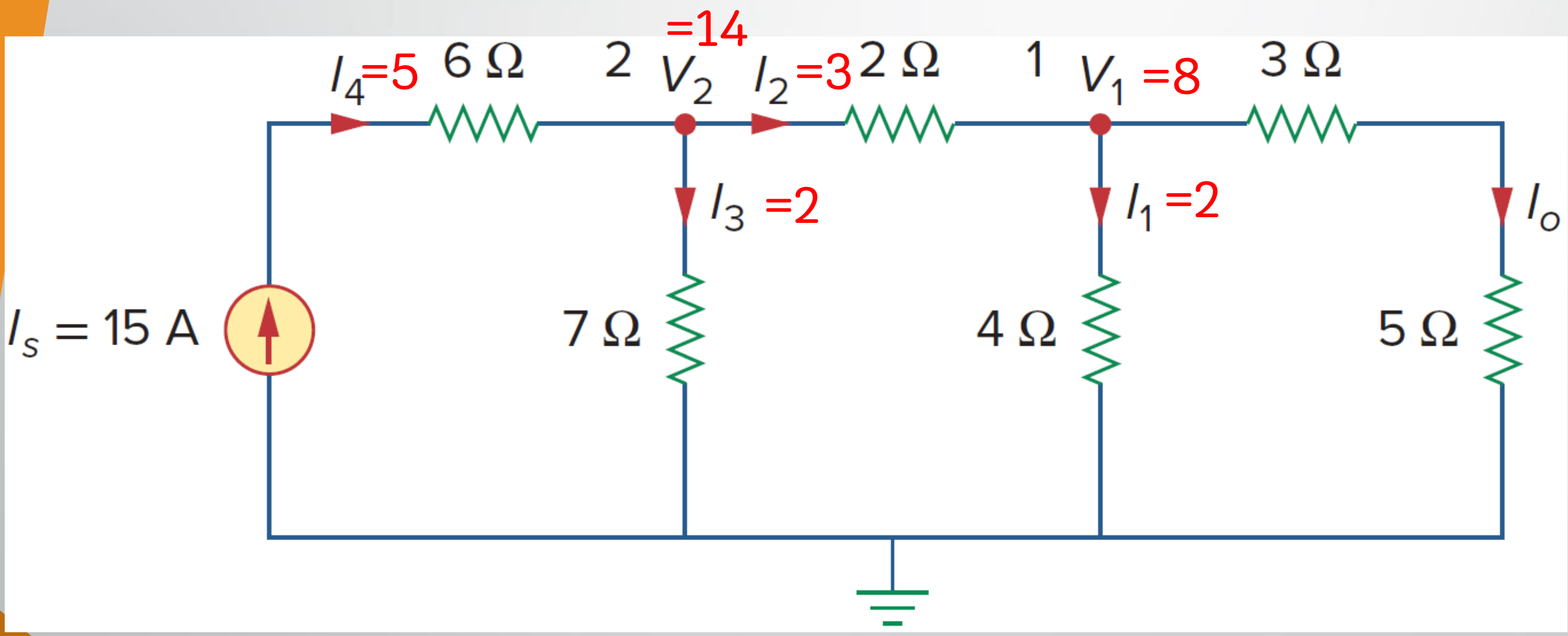
The response to a sum of inputs is the sum of the responses to each input applied separately.

$$v_1 = i_1 R \;,\; v_2 = i_2 R$$

$$\rightarrow \; v = (i_1 + i_2) R = v_1 + v_2$$

4.2 Linearity Property (2)

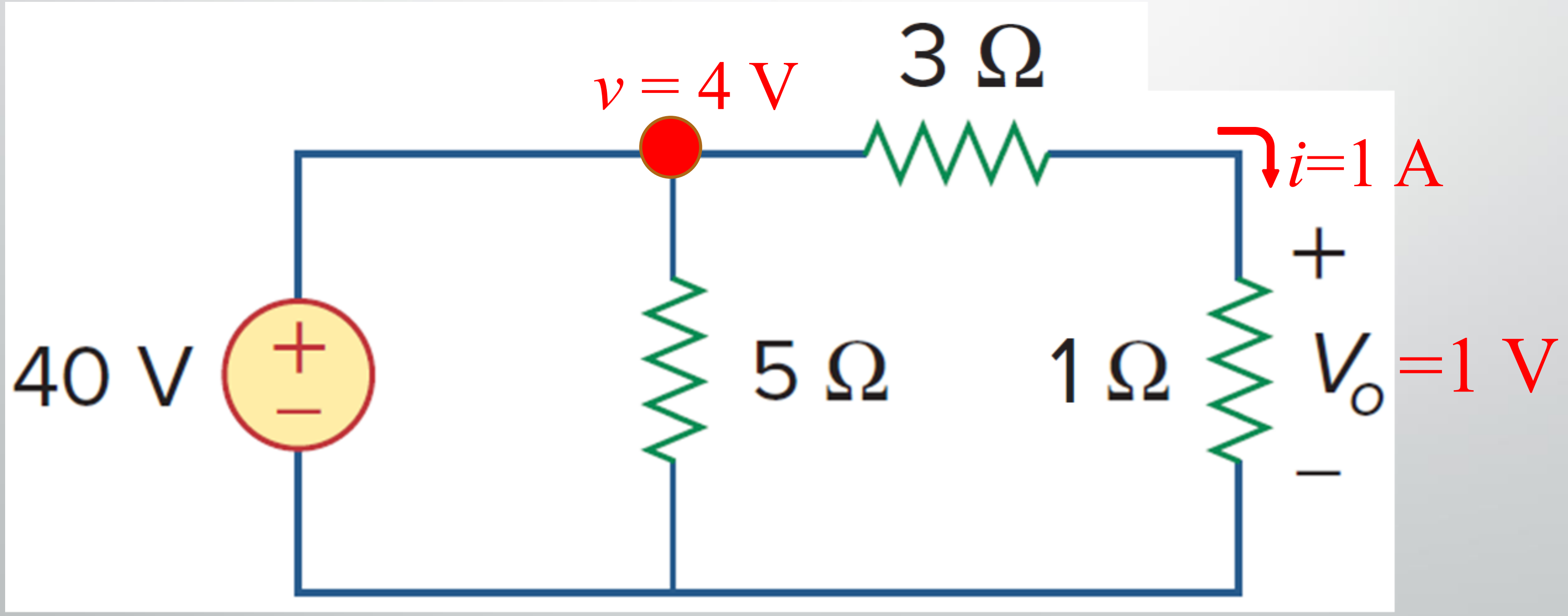
<u>Ex.1</u> By assume $I_o = 1$ A, use linearity to find the actual value of I_o in the circuit shown below.



$I_o = 3$A

4.2 Linearity Property (3)

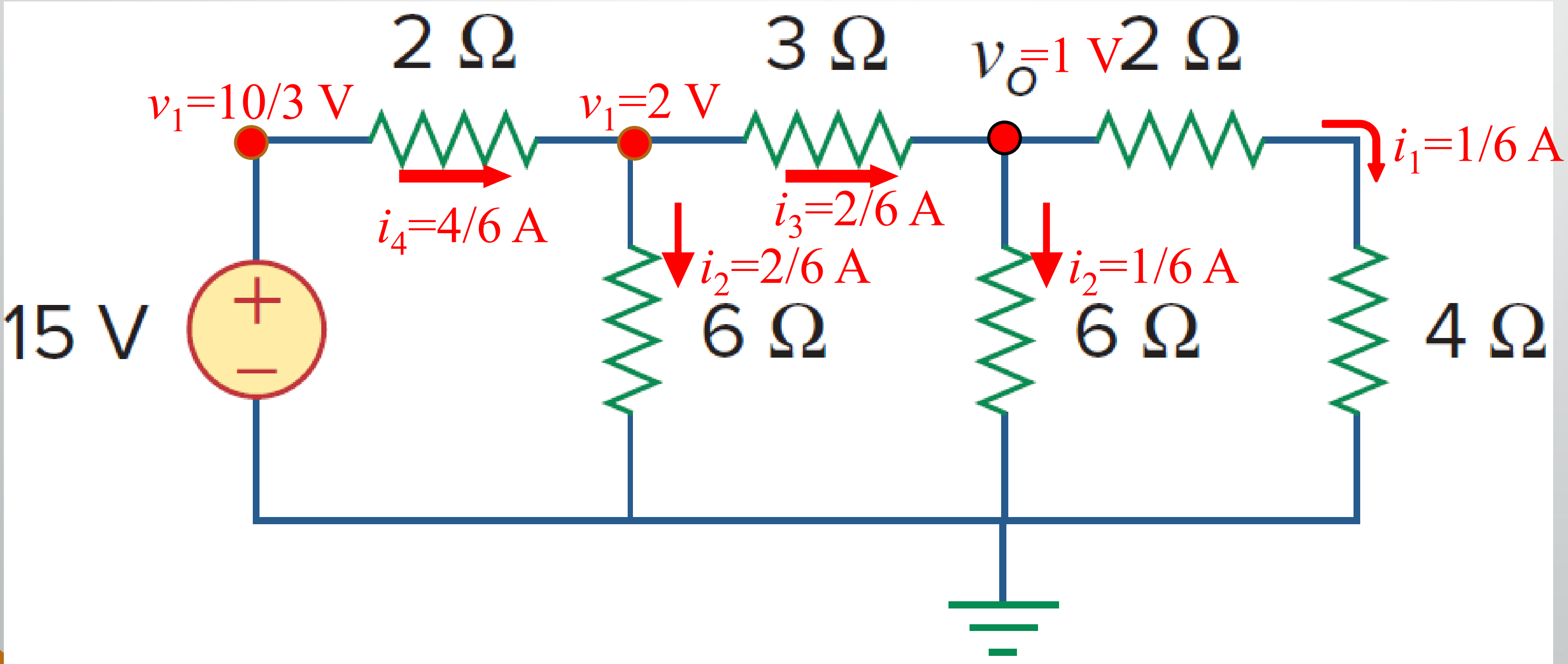
Ex.2 Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig.



$V_o = 40*1/4=10$ V

4.2 Linearity Property (4)

Ex.3 For the circuit in Fig., assume v_o = 1 V, and use linearity to find the actual value of v_o.



v_o = 15*3/10=4.5 V

4.3 Superposition Theorem (1)

It states that the *V* across (or *I* through) an element in a linear circuit is the *algebraic sum* of the voltage across (or currents through) that element due to *EACH independent source acting alone*.
("*V* ที่ตกคล่อมหรือ *I* ที่ไหลผ่านอิลิเมนต์ใดๆ ในวงจรไฟฟ้าเชิงเส้น คือผลรวมพีชคณิตของ *V* หรือ *I* ที่อิลิเมนต์นั้นๆ อันเนื่องมาจากแหล่งกำเนิดแต่ละตัว")

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.
(หลักการซูเปอร์โพซิชั่น ช่วยวิเคราะห์วงจรไฟฟ้าเชิงเส้นที่มีแหล่งกำเนิดอิสระมากกว่า 1 ตัวโดยการแยกคำนวณผลที่เกิดจากแหล่งกำเนิดแต่ละตัวแล้วนำมารวมกัน)

4.3 Superposition Theorem (2)

Steps:

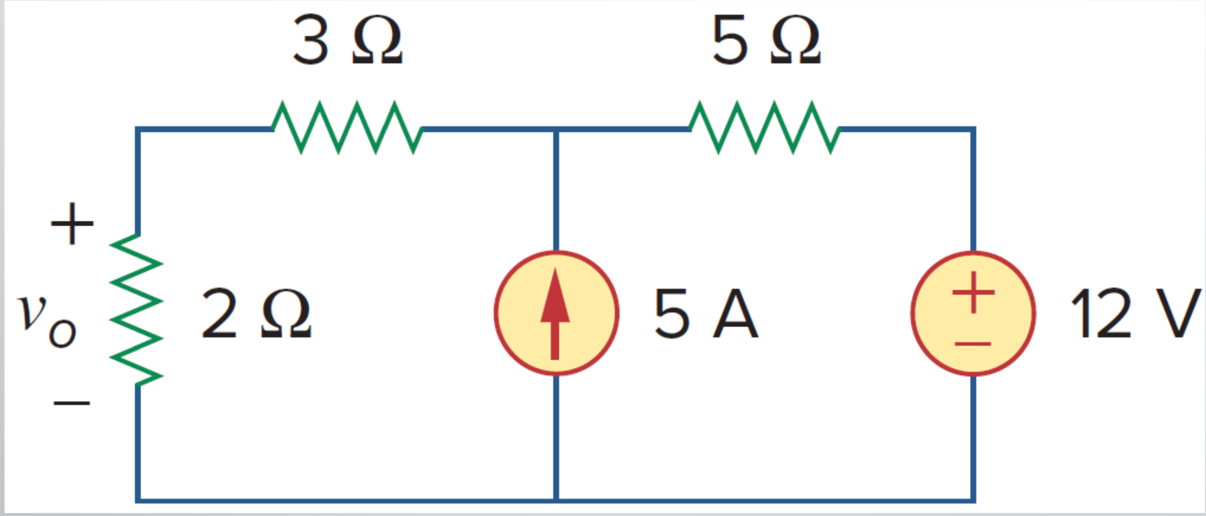
1. Turn off all independent sources except one source. Find the output (*V* or *I*) due to that active source.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

4.3 Superposition Theorem (3)

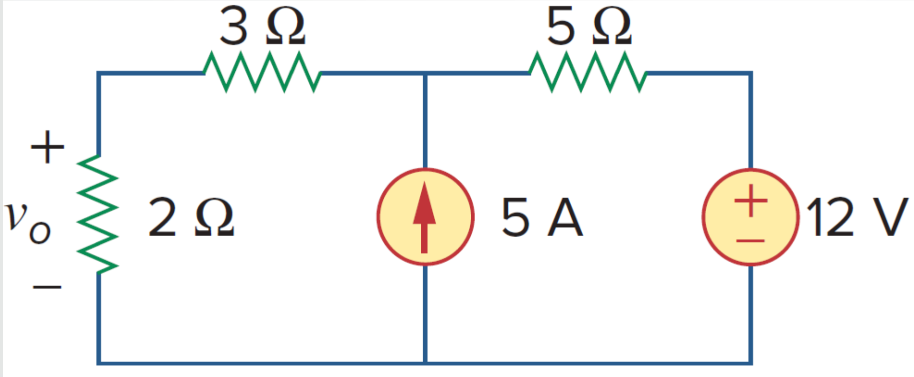
We consider the effects of 5A and 12V one by one, then add the two effects together for final v_o.

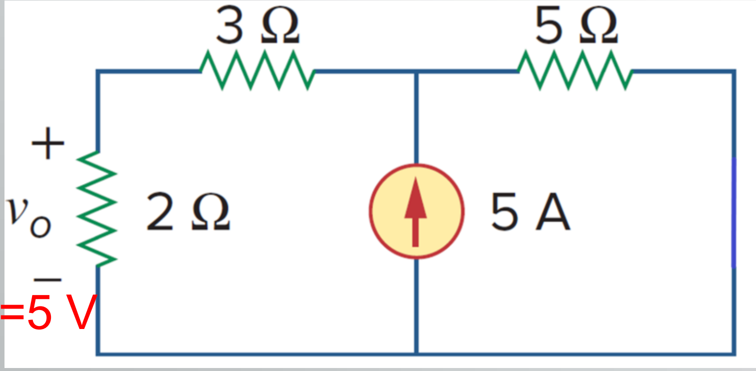


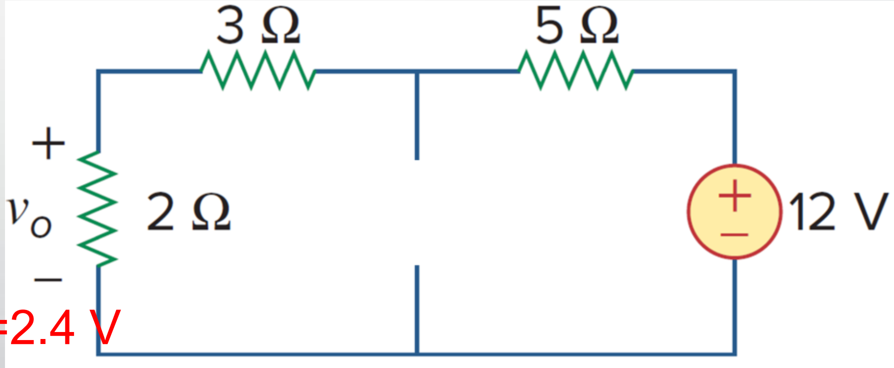
$v_o = 7.4$ V

4.3 Superposition Theorem (4)

Ex.4 Use superposition to find v_x in the circuit below.



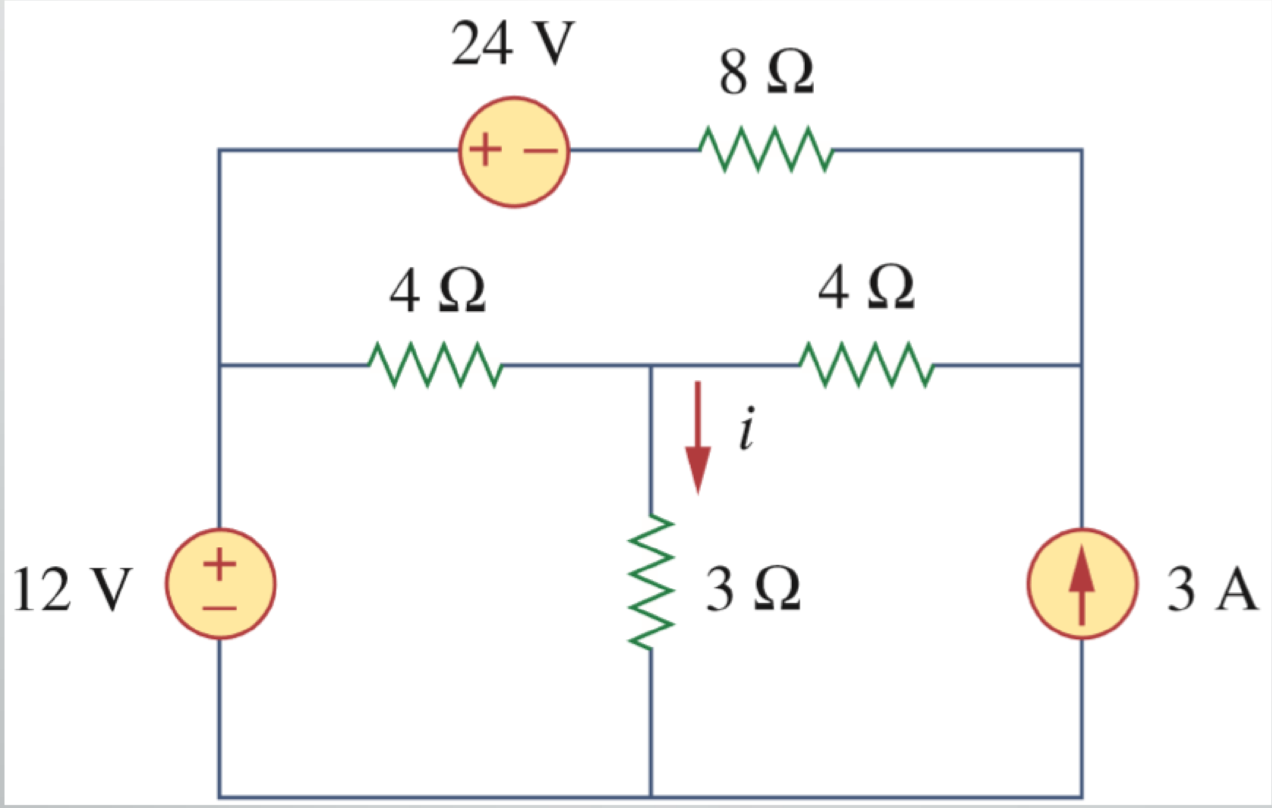




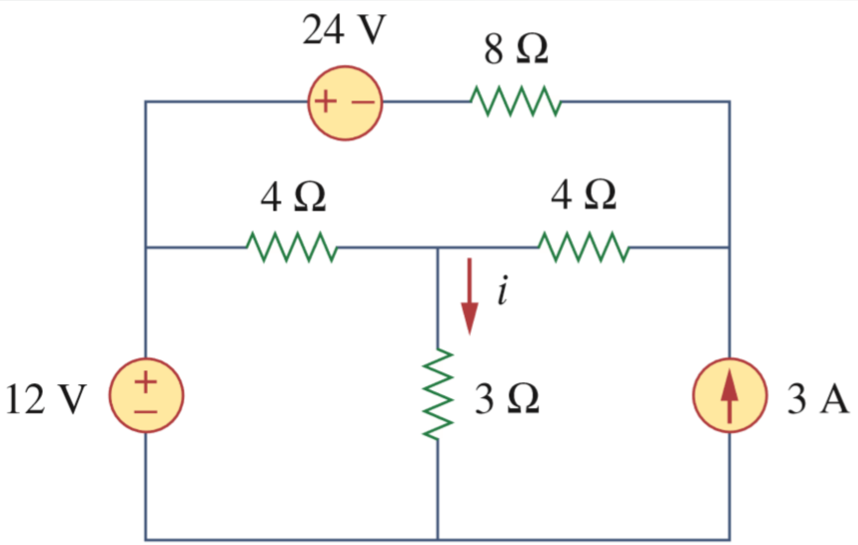
$v_o = 7.4$ V

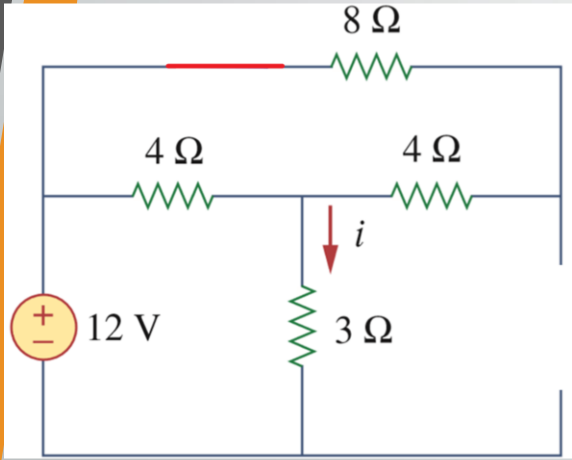
4.3 Superposition Theorem (5)

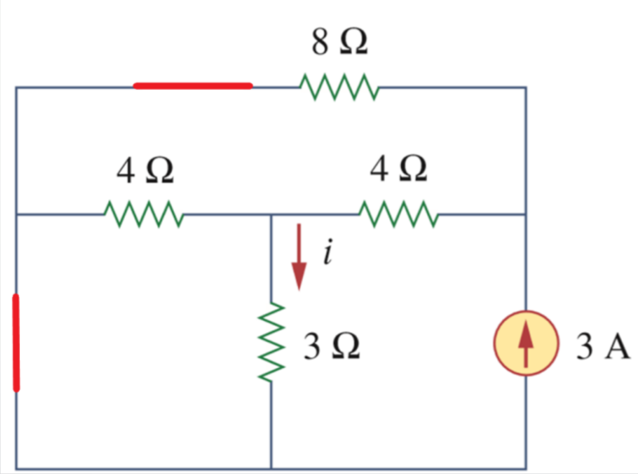
Ex.5 Use superposition to find *i* in the circuit below.

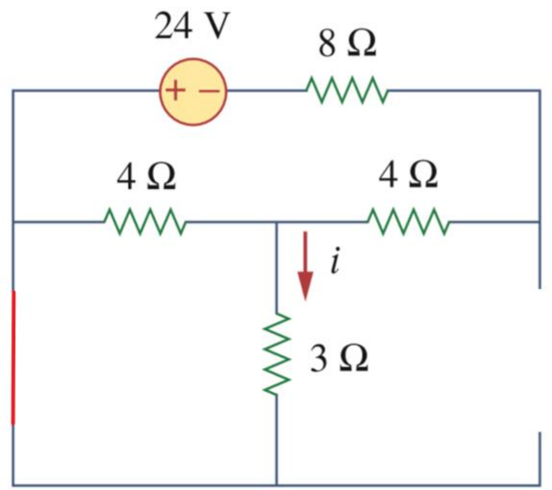


$i = 2A$









$$i_1 = \left(\frac{12}{3 + (4 \parallel 12)}\right) = 2A$$

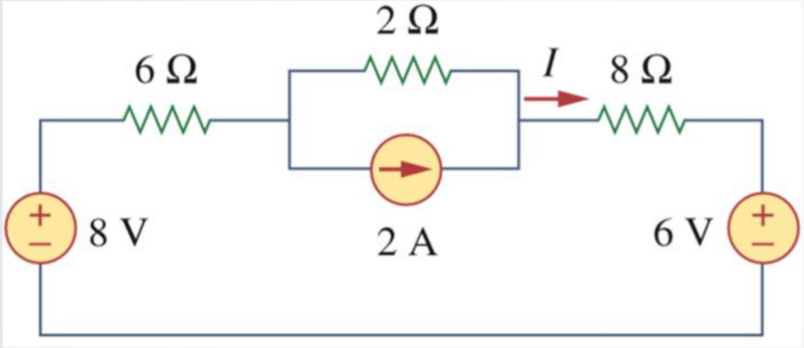
$$i_2 = \left(\frac{8}{8 + (4 + (4 \parallel 3))}\right)\left(\frac{4}{7}\right)3 = 1A$$

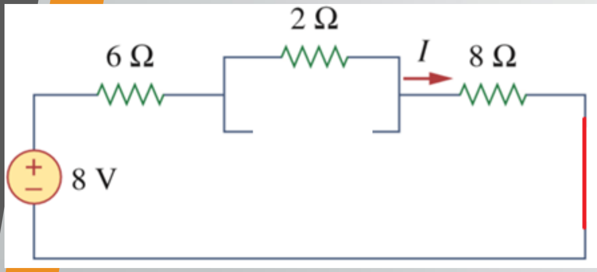
$$i_3 = -\left(\frac{24}{8 + \left(4 + (4 \parallel 3)\right)}\right)\left(\frac{4}{7}\right) = -1A$$

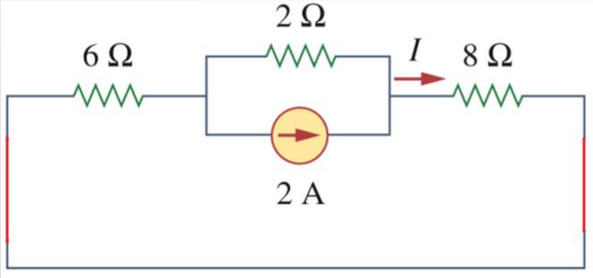
$$i = i_1 + i_2 + i_3 = 2 + 1 - 1 = 2A$$

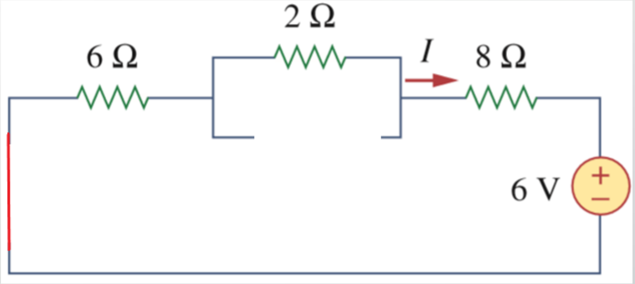
4.3 Superposition Theorem (6)

Ex.6 Use superposition to find *I* in the circuit below.









$I = 375$ mA

4.3 Superposition Theorem (7)

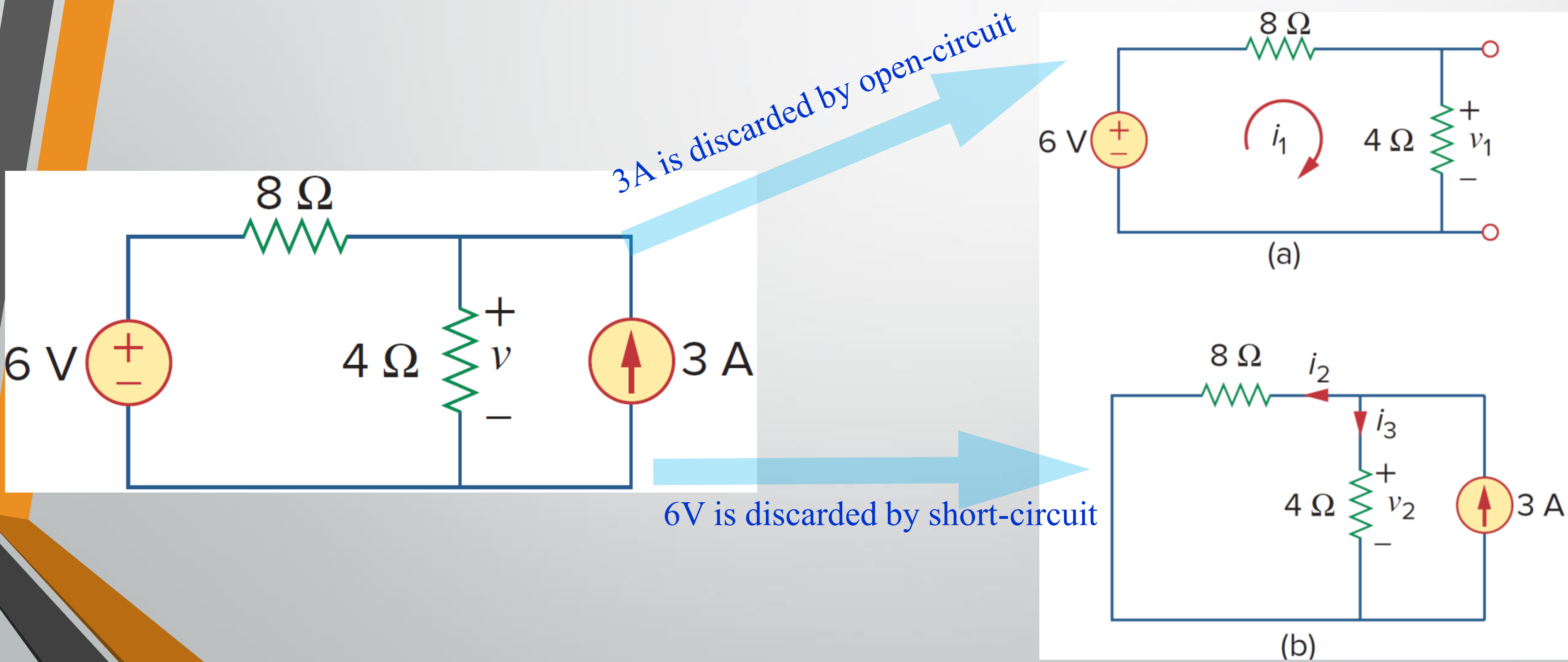
2 things must keep in mind:

1. When we say turn off all other independent sources:
   - Independent *voltage* sources are replaced by 0 V (short circuit) and
   - Independent *current* sources are replaced by 0 A (open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables.

4.3 Superposition Theorem (8)

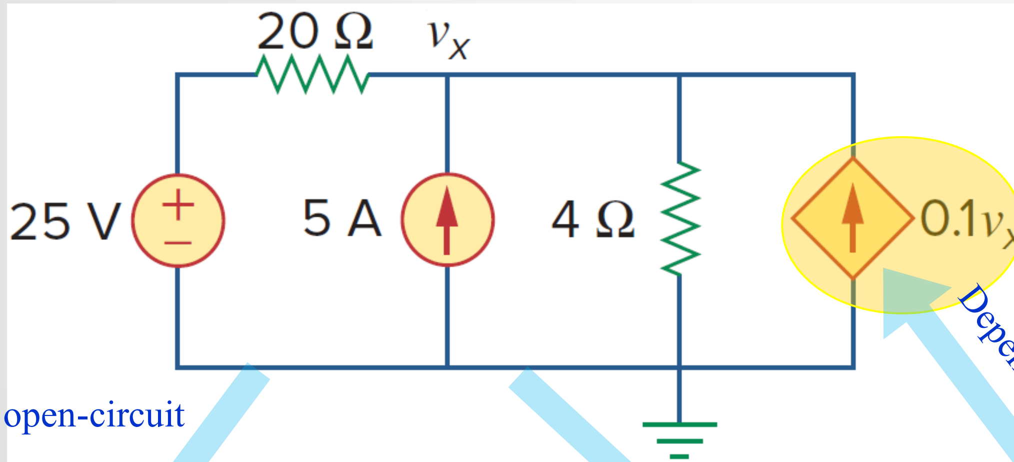
<u>Ex.7</u> Use the superposition theorem to find v in the circuit shown below.



$v = v_1 + v_2$ =2V+8V=10V

4.3 Superposition Theorem (9)

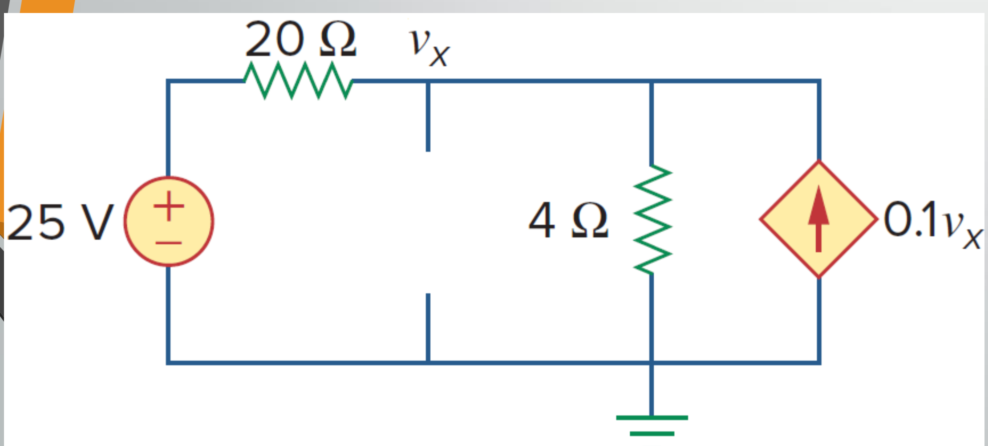
<u>Ex.8</u> Use superposition to find v_x in the circuit below.

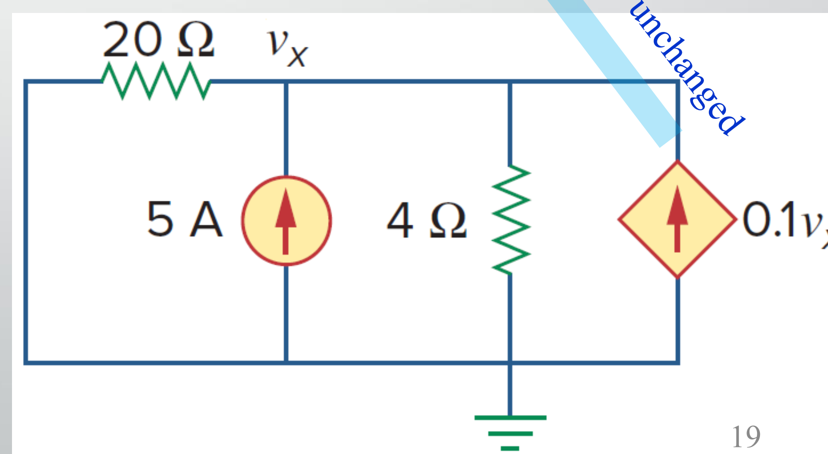


5A is discarded by open-circuit

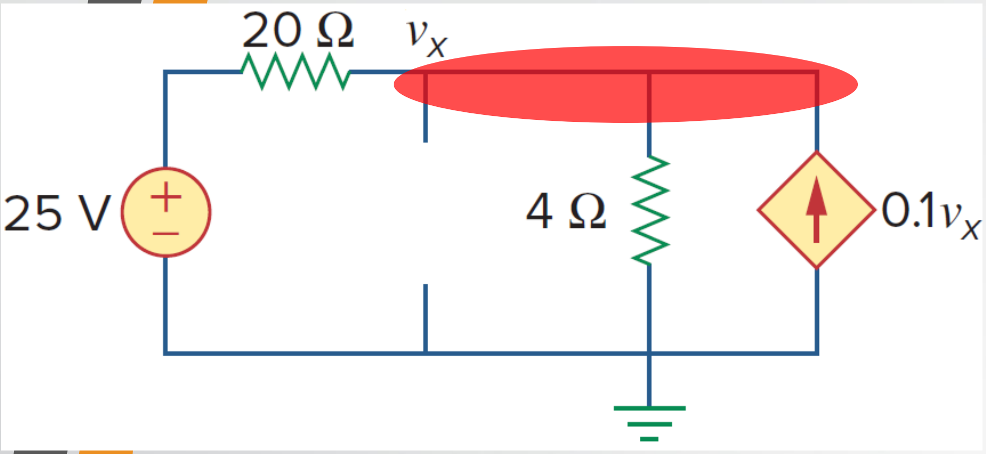
25V is discarded by short-circuit

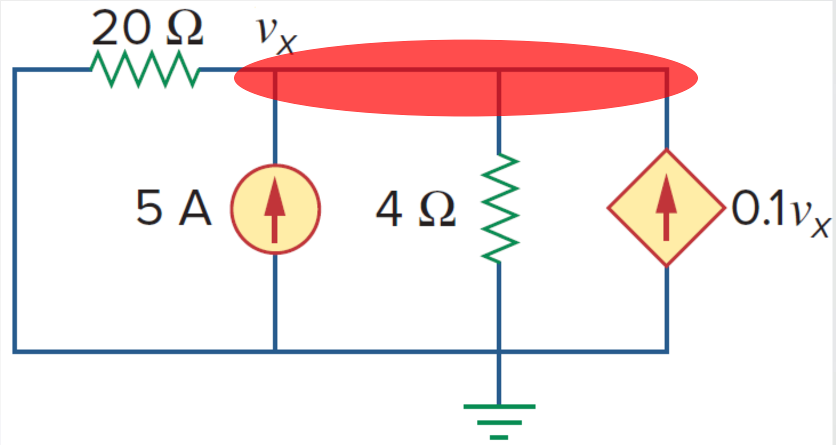
Dependent source keep unchanged





$v_x = 6.25\text{V}+25\text{V}=31.25\text{ V}$





$$\frac{v_x - 25}{20} + \frac{v_x}{4} = 0.1v_x$$

$$v_x - 25 + 5v_x = 2v_x$$

$$v_x = \frac{25}{4} = 6.25\text{V}$$

$$\frac{v_x}{20} + \frac{v_x}{4} = 5 + 0.1v_x$$

$$v_x + 5v_x = 100 + 2v_x$$

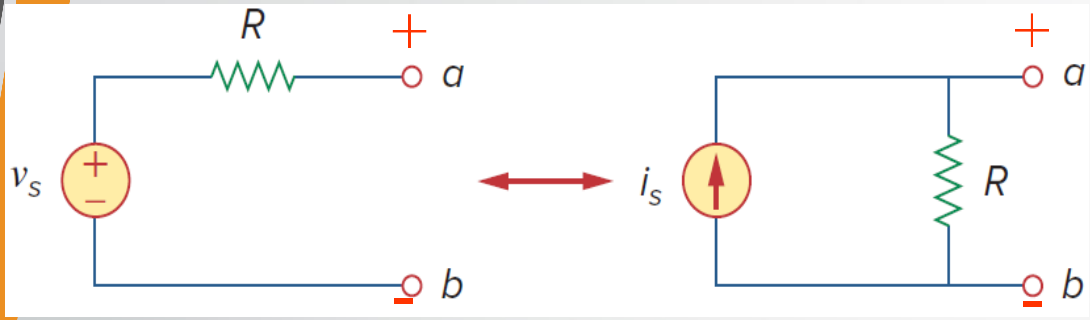
$$v_x = \frac{100}{4} = 25\text{V}$$

4.4 Source Transformation (1)

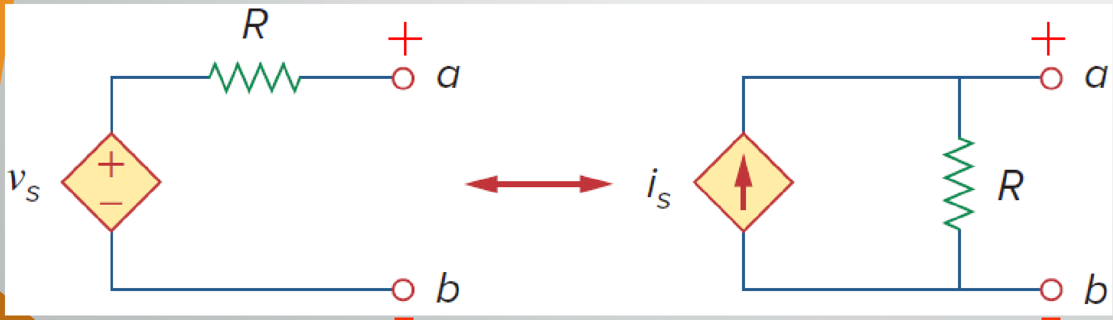
- An equivalent circuit is one whose *V-I* characteristics are identical with the original circuit.

- It is the process of replacing a voltage source V_S in series with a resistor *R* by a current source I_S in parallel with a resistor *R*, or vice versa.

4.4 Source Transformation (2)



(a) Independent source transform



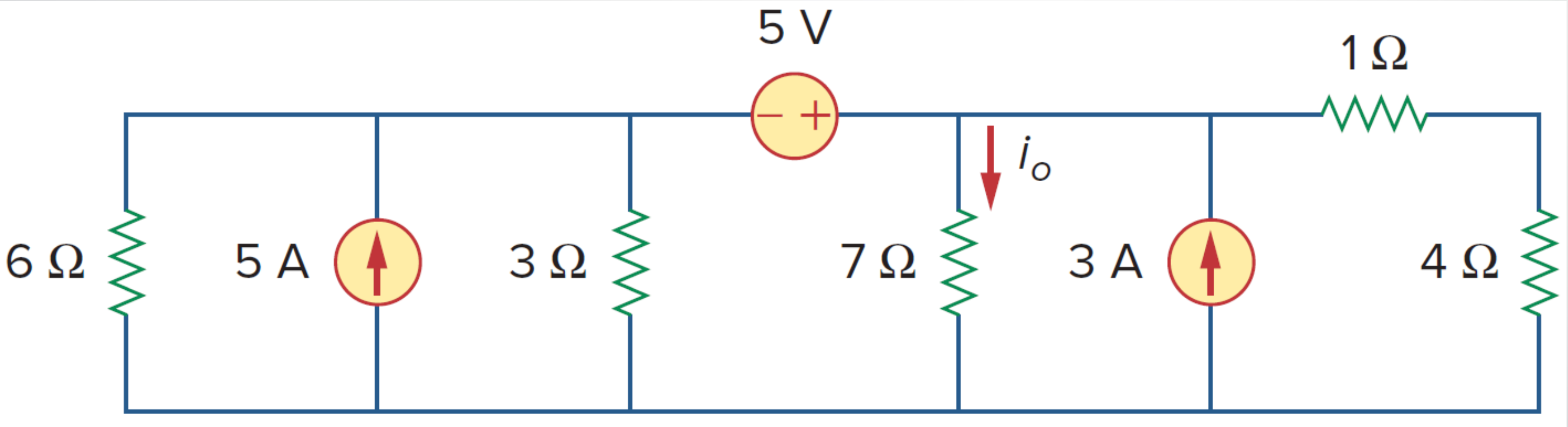
(b) Dependent source transform

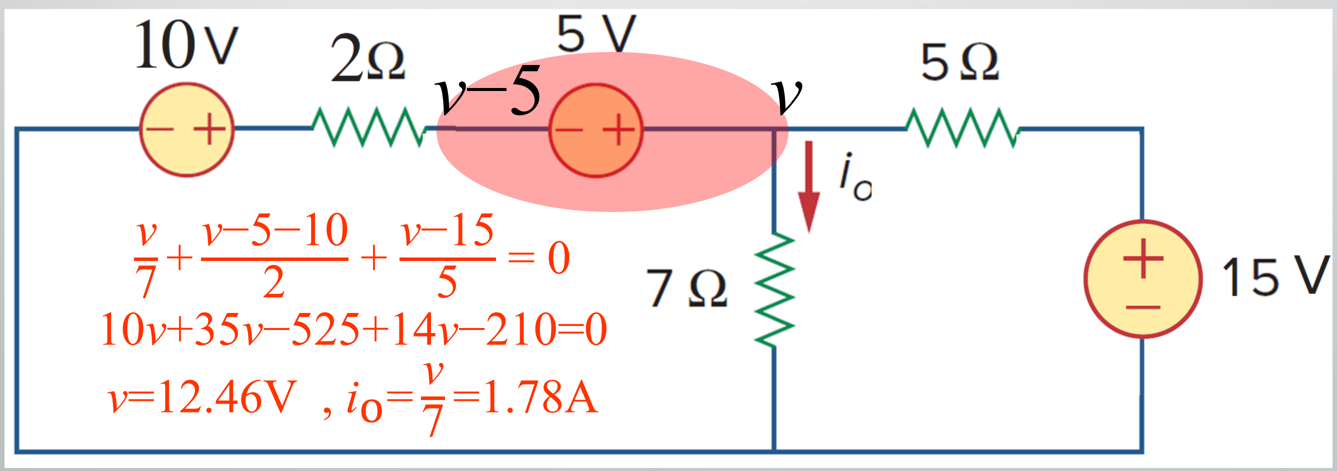
- The arrow of the current source is directed toward the positive (+) terminal of the voltage source.

- The transformation is not possible when $R = 0$ for voltage source and $R = \infty$ for current source.

4.4 Source Transformation (3)

Ex.9 Find i_o in the circuit shown below using source transformation.





$$\frac{v}{7}+\frac{v-5-10}{2}+\frac{v-15}{5}=0$$

$$10v+35v-525+14v-210=0$$

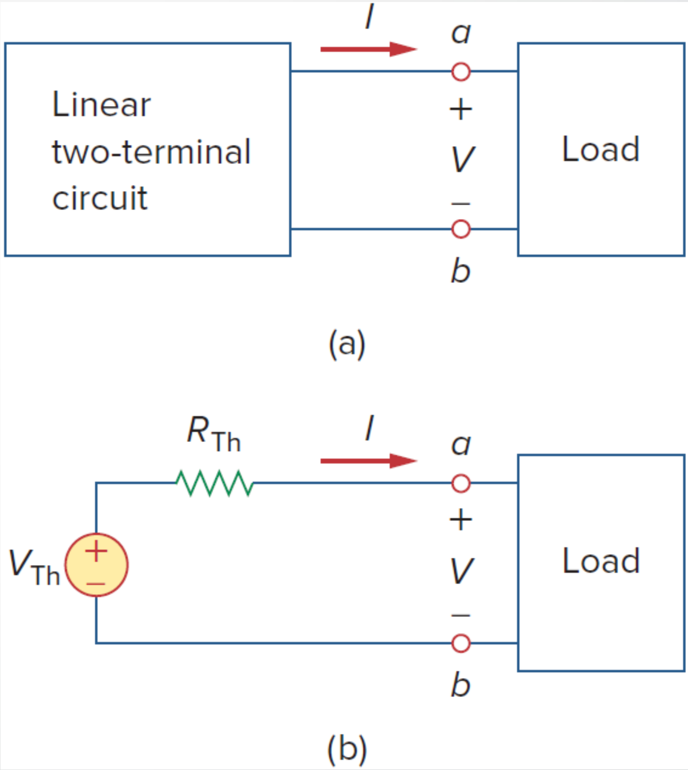
$$v=12.46\text{V}\ ,\ i_o=\frac{v}{7}=1.78\text{A}$$

4.5 Thevenin's Theorem (1)

It states that a linear two-terminal circuit
(a) can be replaced by an equivalent circuit
(b) (b) consisting of a voltage source V_{Th} in series with a resistor R_{Th}
, where

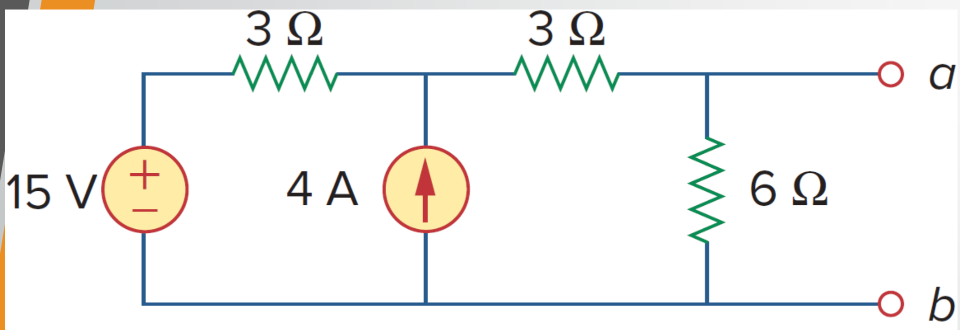
- V_{Th} is the open-circuit voltage at the terminals.

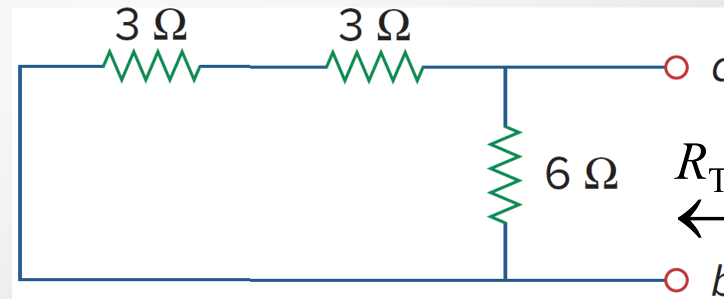
- R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

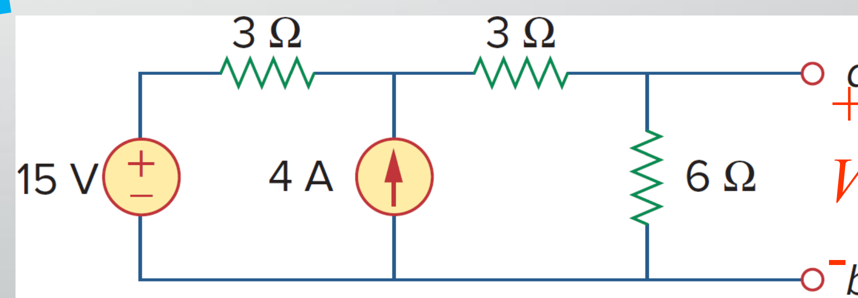


4.5 Thevenin’s Theorem (2)

Ex.10 Using Thevenin’s theorem, find the equivalent circuit to the left of the terminals in the circuit shown below.



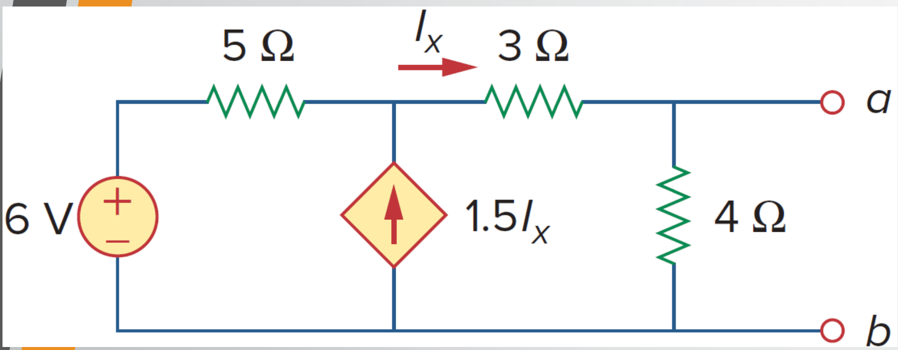


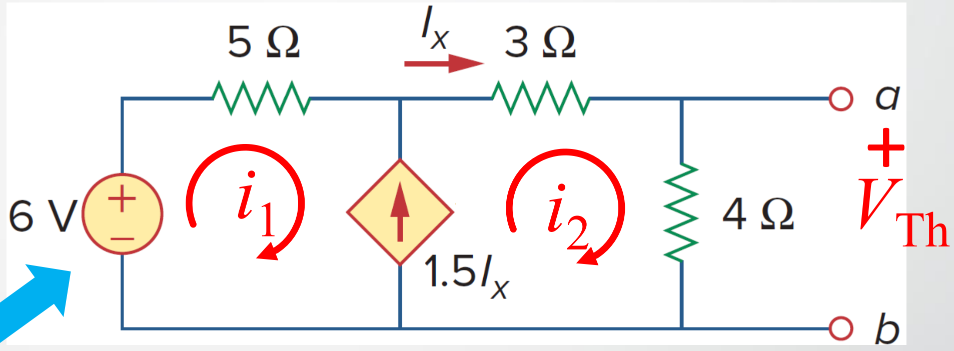


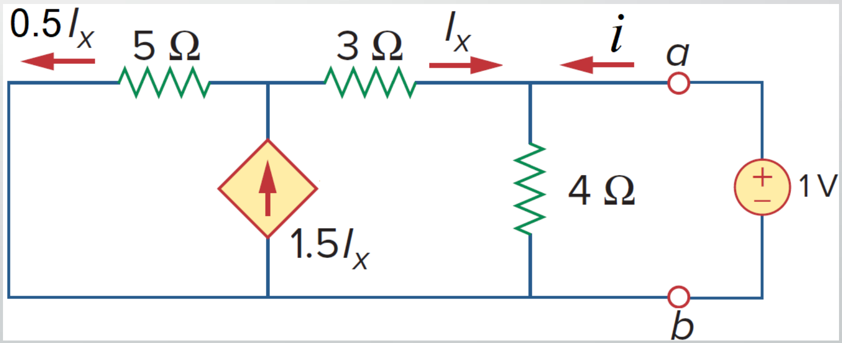
$V_{Th} = 13.5\text{V}, R_{Th} = 3\Omega$

4.5 Thevenin's Theorem (3)

Ex.11 Find the Thevenin equivalent circuit of the circuit shown below to the left of the terminals.







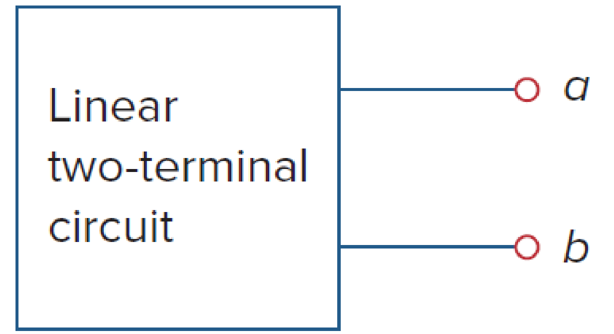
V_{Th} = 16/3=5.333 V, R_{Th} =4/9=0.444 Ω

4.6 Norton's Theorem (1)

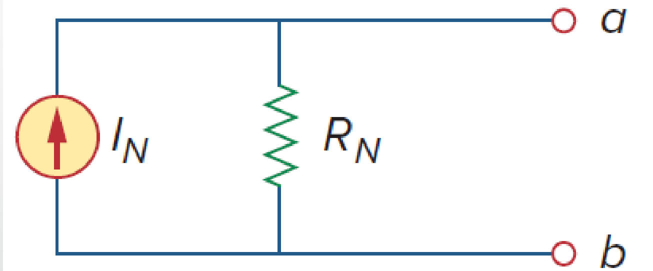
It states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where

- I_N is the short circuit current through the terminals.
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

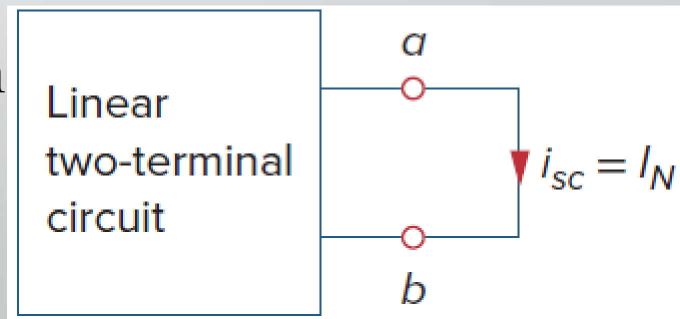
The Thevenin's and Norton equivalent circuits are related by a source transformation



(a) Original circuit



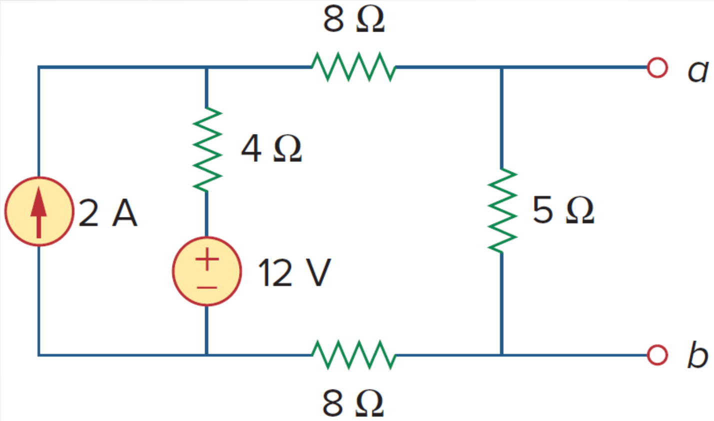
(b) Norton equivalent circuit.

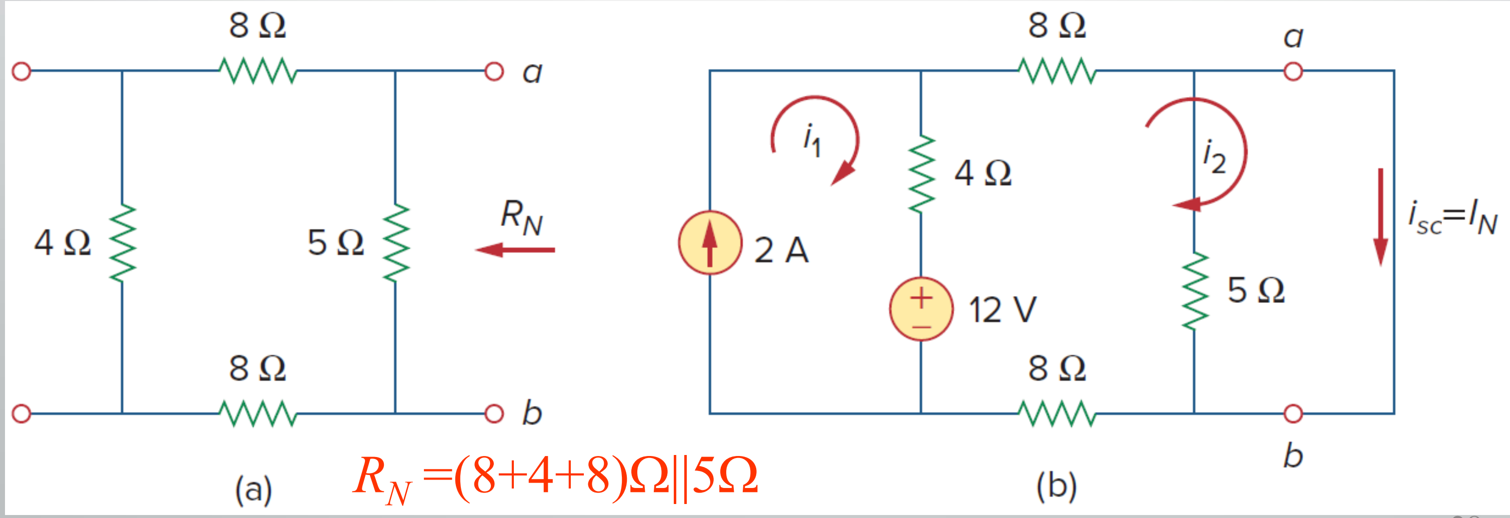


(c) Finding Norton current I_N.

4.6 Norton's Theorem (2)

Ex.12 Find the Norton equivalent circuit of the circuit shown below.





$R_N = 4\Omega$, $I_N = 1$A.

4.6 Norton's Theorem (3)

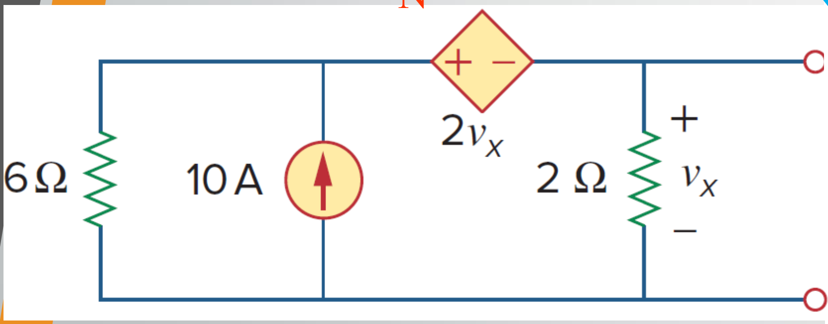
Ex.13 Find the Norton equivalent circuit of the circuit shown below.

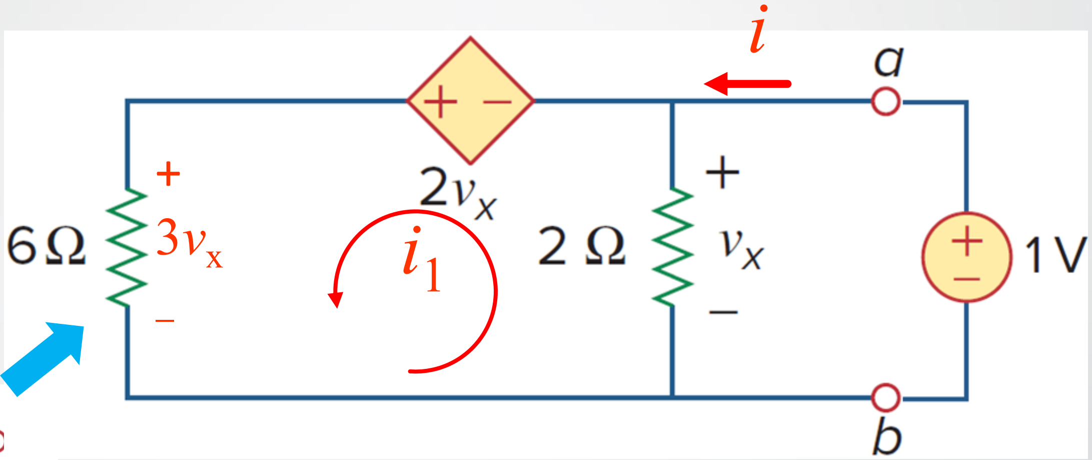
$v_x = 1V$

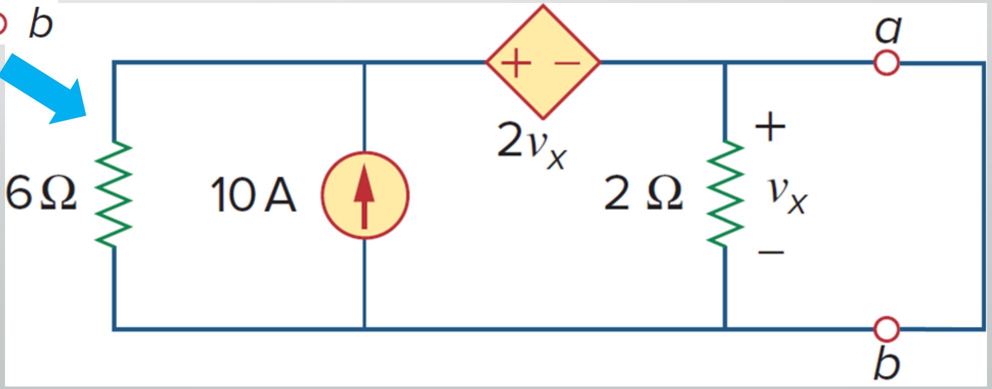
$i_1 = 3v_x/6 = 0.5A$

$i = i_1 + v_x/2 = 1A$

$R_N = 1/i = 1\Omega$





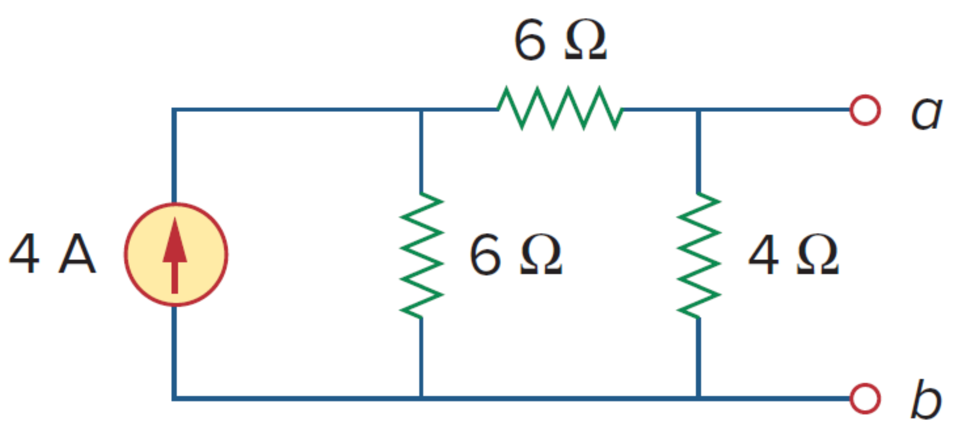


I_{sc}

$v_x = 0V$

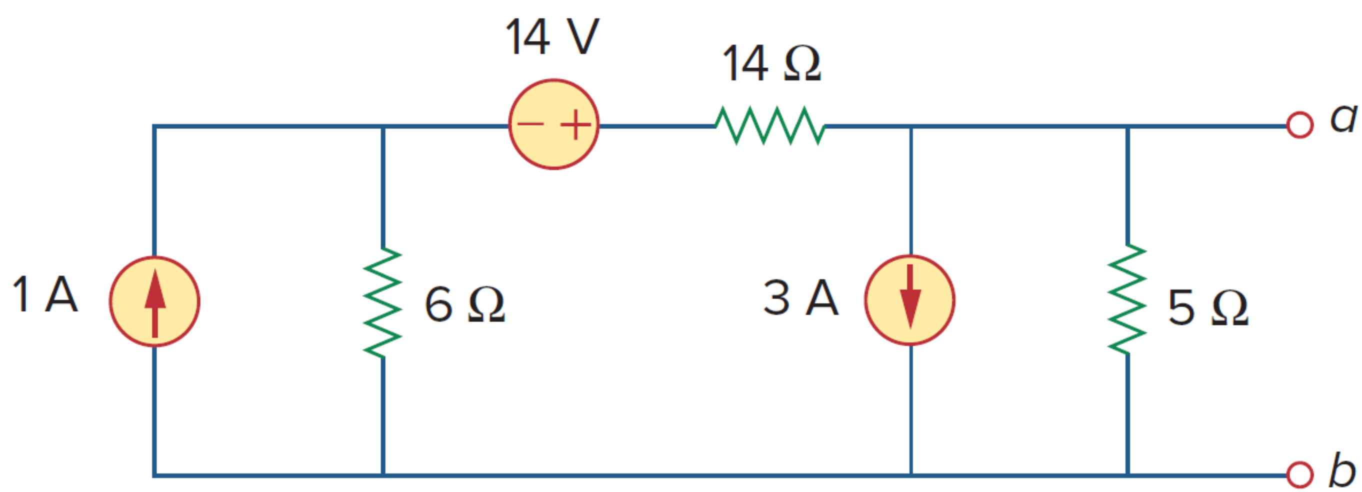
$I_{sc} = 10A$

4.45 Find the Thevenin equivalent of the circuit in Fig. 4.112 as seen by looking into terminals *a* and *b*.



3 Ω, 6 V

4.41 Find the Thevenin and Norton equivalents at terminals *a*-*b* of the circuit shown in Fig. 4.108.

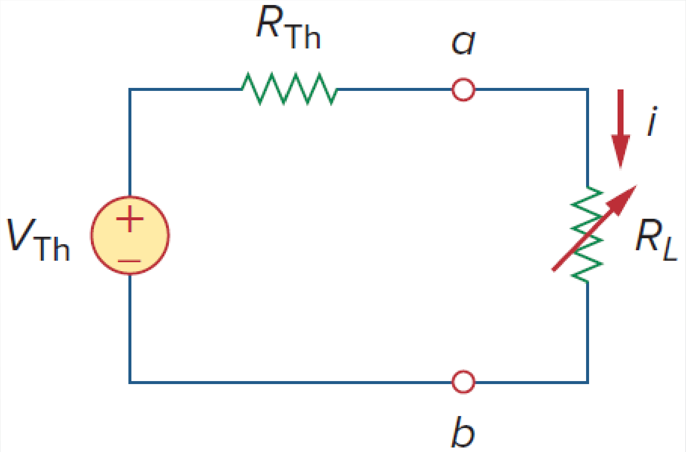


4 Ω, −8 V, −2 A

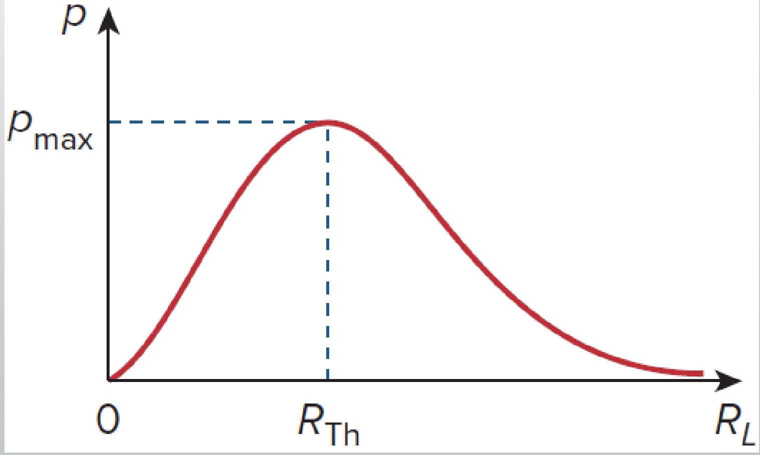
4.7 Maximum Power Transfer (1)

Maximum power is transferred to the load when the load resistance R_L equals the Thevenin resistance R_{TH} as seen from the load ($R_L = R_{TH}$).

$$P = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 R_L$$



$$R_{TH} = R_L, \qquad P_{max} = \frac{V_{TH}^2}{4R_L}$$



Power delivered to the load as a function of R_L

4.7 Maximum Power Transfer (2)

$$P = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\frac{dP}{dR_L} = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 + 2\left(\frac{V_{Th}}{R_{Th} + R_L}\right)\left(\frac{-V_{Th}}{(R_{Th} + R_L)^2}\right)R_L = 0$$

$$(R_{Th} + R_L) - 2R_L = 0$$

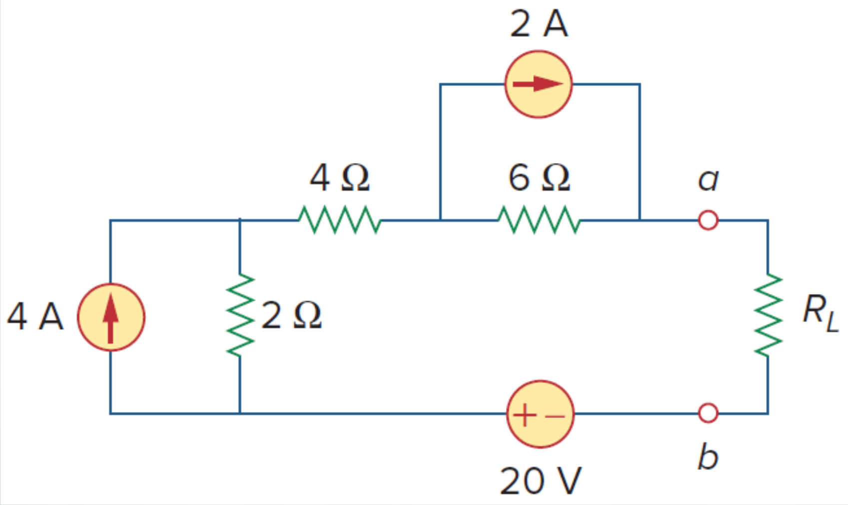
$$R_{Th} = R_L$$

4.7 Maximum Power Transfer (3)

• <u>Ex.14</u>

a) Find R_L for maximum power deliverable to R_L

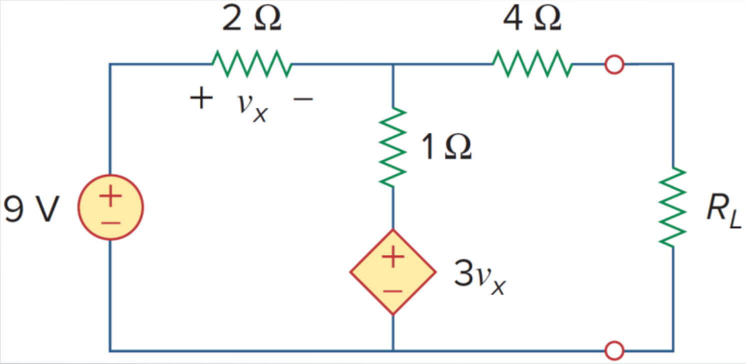
b) Determine that maximum power.

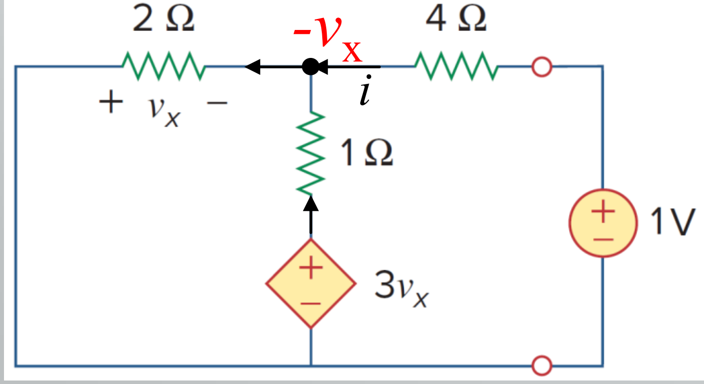


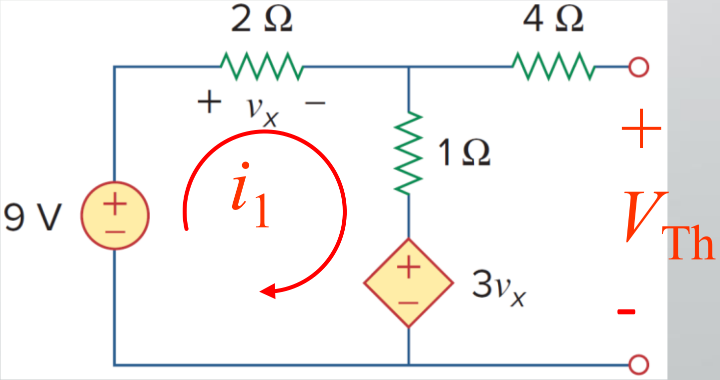
$R_L = 12\ \Omega$, $V_{TH} = 40$ V , $P_{max} = 33.33$W

4.7 Maximum Power Transfer (4)

Ex.15 Determine the value of R_L that will draw the maximum power from the rest of the circuit shown below. Calculate the maximum power.

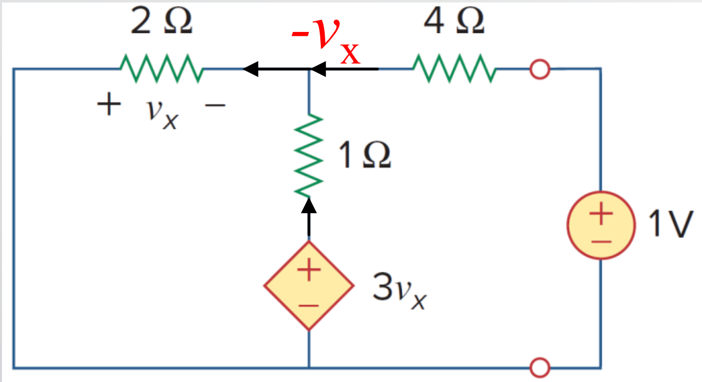


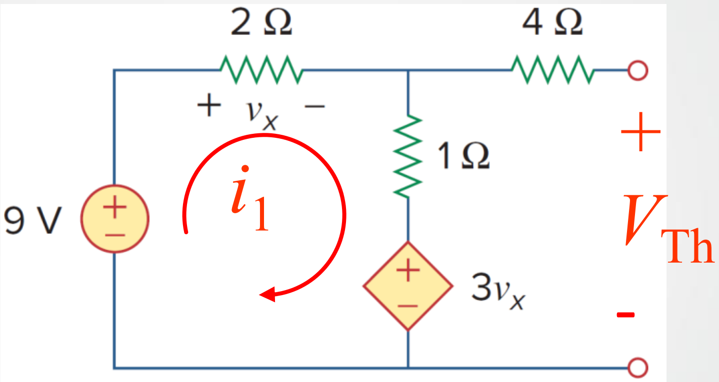




$R_L = 4.22\Omega$, $P_m = 2.901\text{W}$

4.7 Maximum Power Transfer (5)





$$i = \frac{1-(-v_x)}{4} = \frac{1+v_x}{4}$$

$$\frac{1+v_x}{4} + \frac{3v_x-(-v_x)}{1} = \frac{-v_x}{2}$$

$$v_x = -\frac{1}{19},\ i = \frac{9}{38},\ R_{Th} = \frac{38}{9} = 4.22\Omega$$

$$v_x = 2i_1$$

$$-9 + v_x + 1i_1 + 3v_x = 0$$

$$i_1 = 1\ \text{A},\ V_{Th} = 7\ \text{V}$$

$$P_{max} = \frac{V_{Th}^{\ 2}}{4R_{Th}}$$

$$P_{max} = 2.901\ \text{W}$$