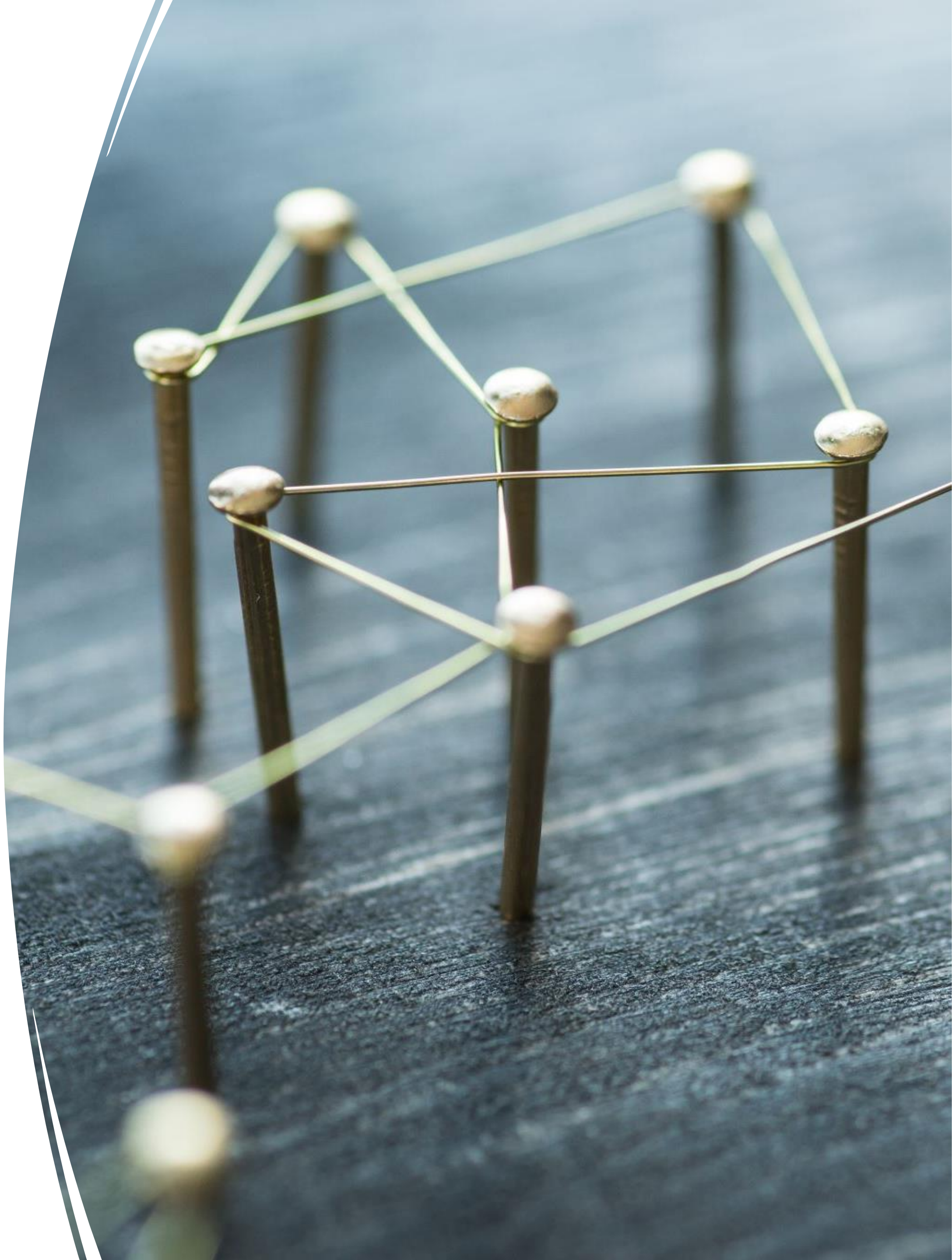
# Electrical Engineering 1
# 12026105

## Chapter 3

## Methods of Analysis

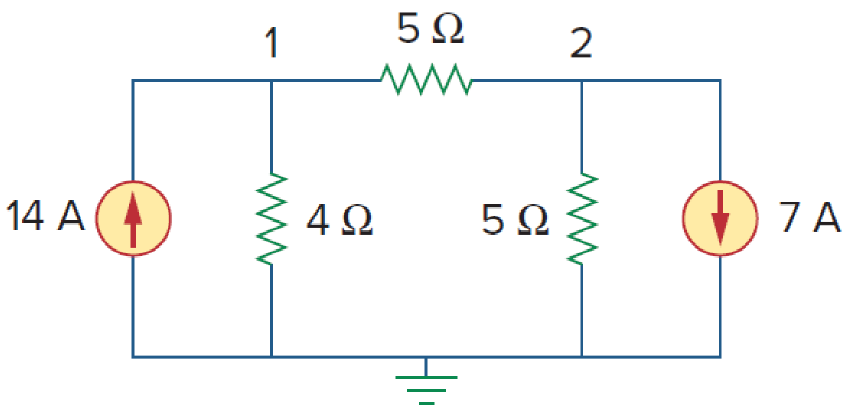
# Methods of Analysis - Chapter 3

- Introduction
- Nodal analysis.
- Nodal analysis with voltage sources.
- Mesh analysis.
- Mesh analysis with current sources.
- Nodal and mesh analysis by inspection.
- Nodal versus mesh analysis.



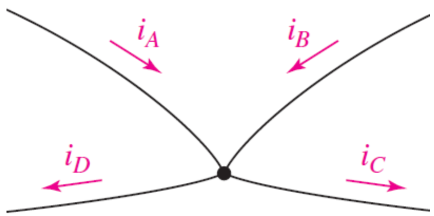
# 3.1 Introduction (1)

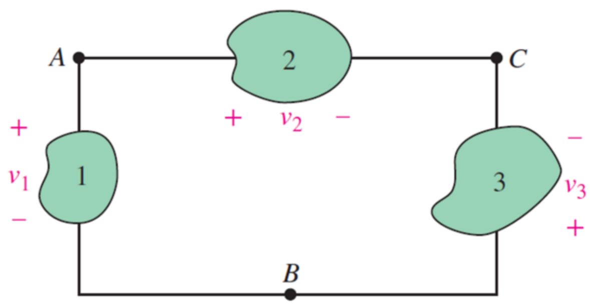
- If you are given the following circuit, how can you determine
  - Voltage across each resistor.
  - Current through each resistor.
  - Power generated by each current source, etc.

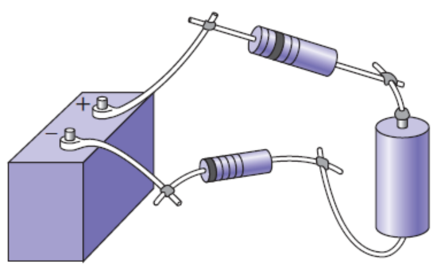



What are the things which we need to know to determine the answers?

# 3.1 Introduction (2)




KIRCHHOFF'S CURRENT LAWS (KCL)

KIRCHHOFF'S VOLTAGE LAWS (KVL)

OHM'S LAW

How should we apply these laws to determine the answers?

จะใช้กฎเหล่านี้หาคำตอบของวงจรไฟฟ้าได้อย่าง?

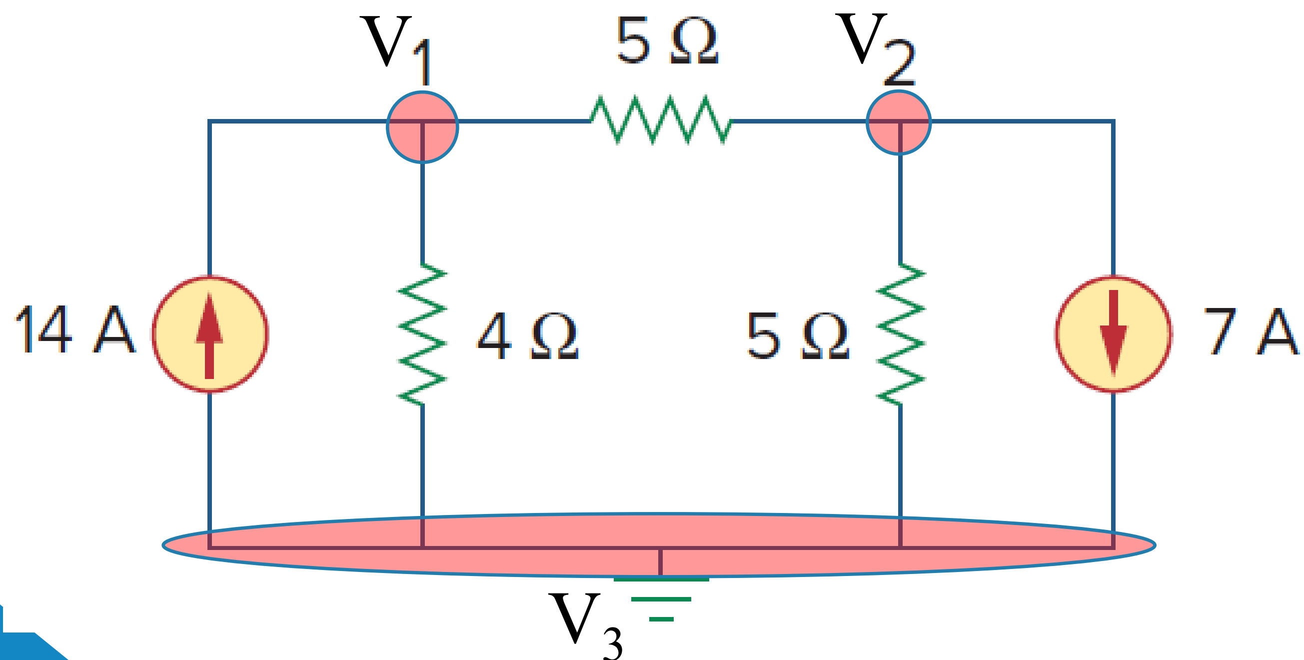
# 3.2 Nodal Analysis (1)

Apply these laws to develop two powerful techniques for circuit analysis:

Nodal analysis, based on a systematic application of KCL&Ω, and

Mesh analysis, based on a systematic application of KVL &Ω.

<u>Ex.</u> Analyzing circuits using *<u>nodal voltages</u>* as circuit variables.



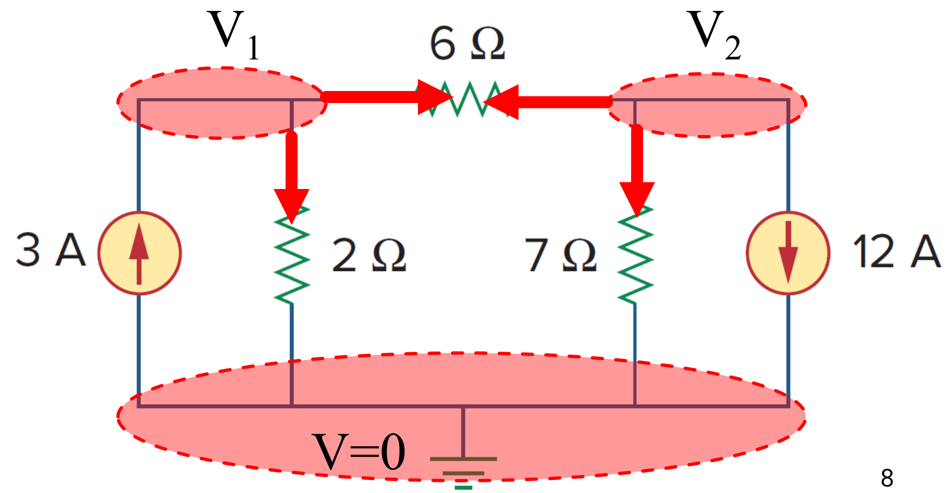
# 3.2 Nodal Analysis (2)

Steps:

1. Select a node as the reference node ($V=0$). Assign voltages $V_1$, $V_2$,… to the remaining nodes.
2. Apply KCL to each of the nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

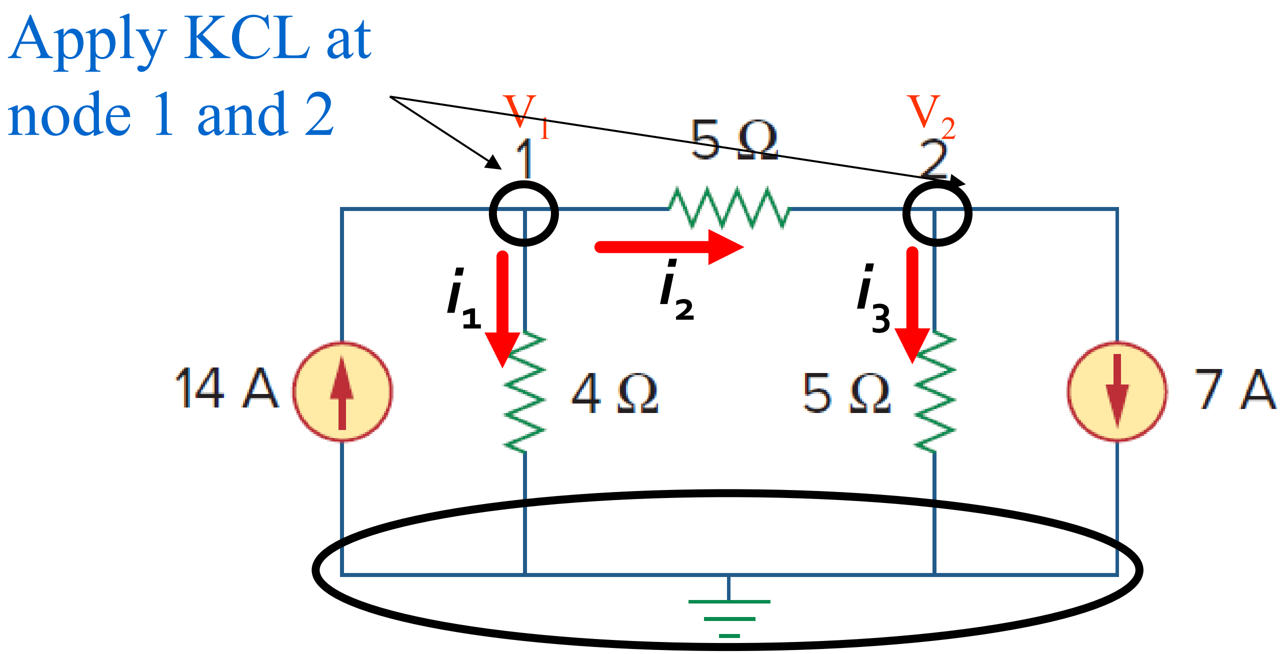
Apply KCL at node$V_1$ : $3=\frac{V_1}{2}+\frac{V_1-V_2}{6}$

Apply KCL at node$V_2$ : $12+\frac{V_2}{7}+\frac{V_2-V_1}{6}=0$



# 3.2 Nodal Analysis (3)

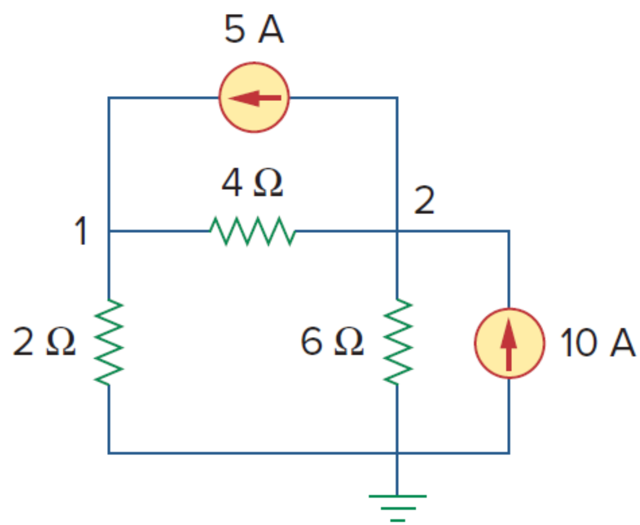
Ex.1 Circuit independent current source only




$V_1 = 30$ V, $V_2 = -2.5$ V

# 3.2 Nodal Analysis (4)

Ex.2 Calculate the node voltages in the circuit




At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \quad \Rightarrow \quad 3v_1 - v_2 = 20$$

At node 2, we do the same thing and get

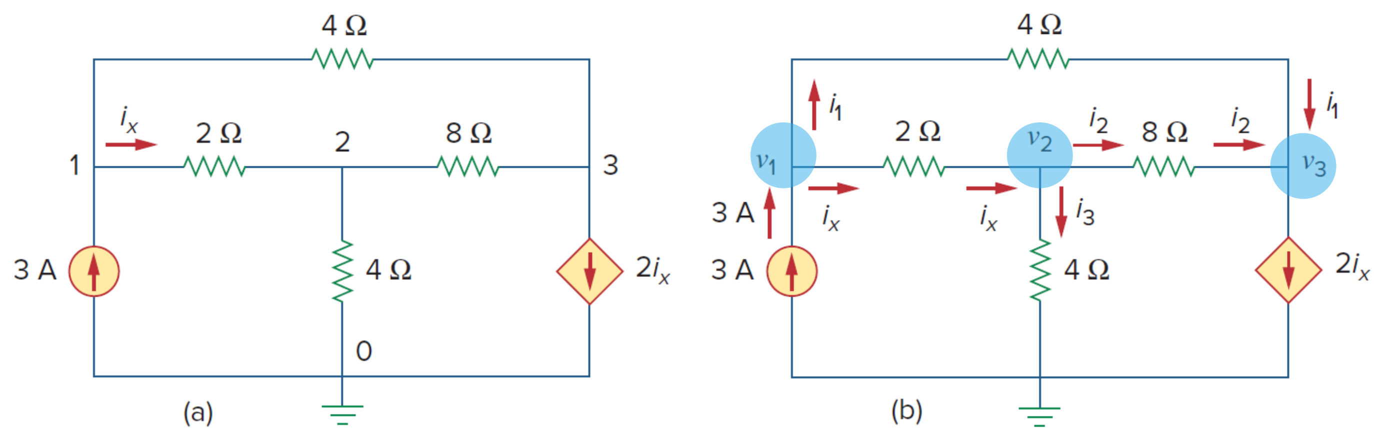
$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6} \quad \Rightarrow \quad -3v_1 + 5v_2 = 60$$



$v_1 = 13.33$ V, $v_2 = 20$ V

# 3.2 Nodal Analysis (5)

## Ex.3 Determine the voltages at the nodes in Fig.




At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \quad \Rightarrow \quad 3v_1 - 2v_2 - v_3 = 12$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \quad \Rightarrow \quad -4v_1 + 7v_2 - v_3 = 0$$

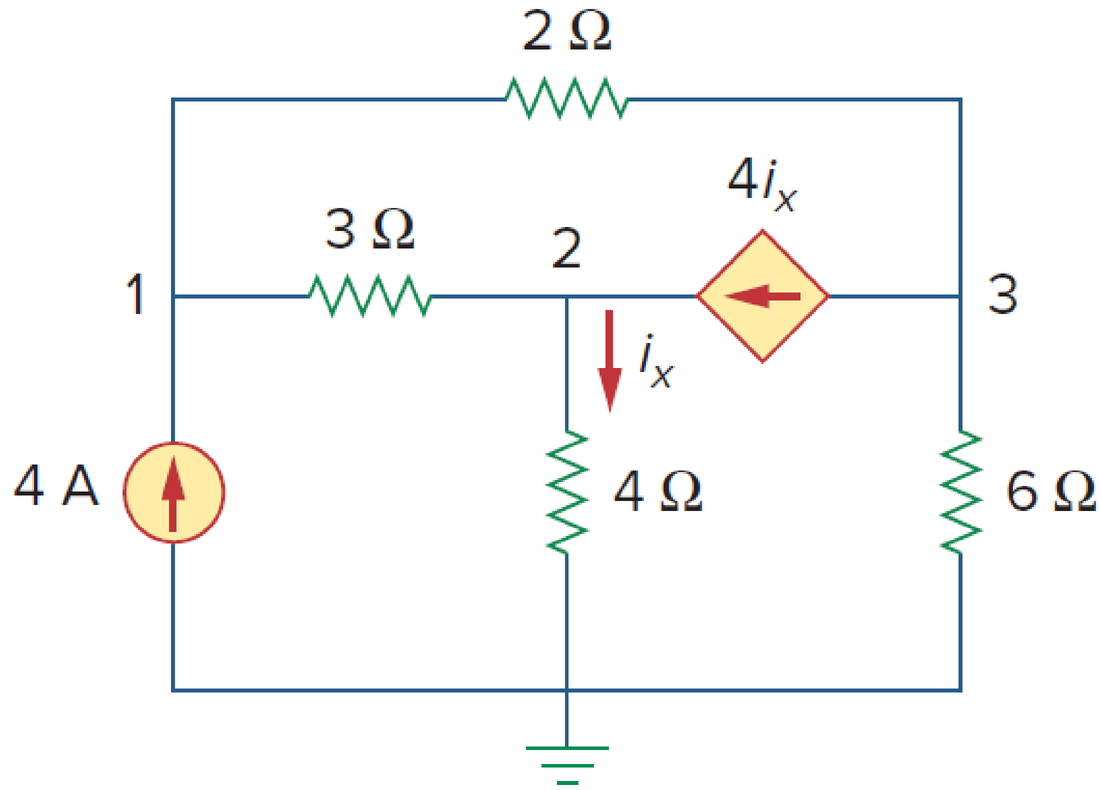
At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2} \quad \Rightarrow \quad 2v_1 - 3v_2 + v_3 = 0$$

$v_1 = 4.8\text{V}$, $v_2 = 2.4\text{V}$, $v_3 = -2.4\text{V}$

# 3.2 Nodal Analysis (6)

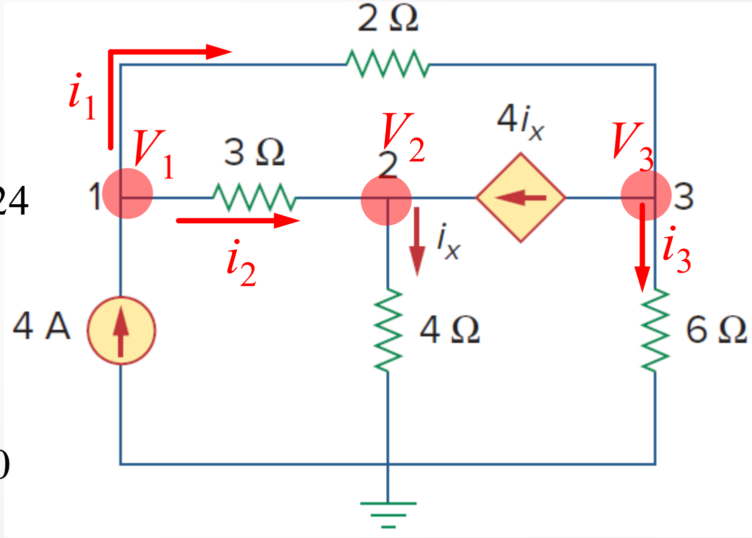
Ex.4 Find the voltages at the three non-reference nodes in the circuit of Fig.




$v_1 = 32V$, $v_2 = -25.6V$, $v_3 = 62.4V$

# 3.2 Nodal Analysis (6)

Ex.4 Find the voltages at the three non-reference nodes in the circuit of Fig.




ที่โหนด 1 : $4 = i_1 + i_2\ ;\ 4 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3} \Rightarrow 5V_1 - 2V_2 - 3V_3 = 24$

ที่โหนด 2 : $i_2 + 4i_x = i_x ; \frac{V_1 - V_2}{3} = -3 * \frac{V_2}{4} \Rightarrow 4V_1 + 5V_2 = 0$

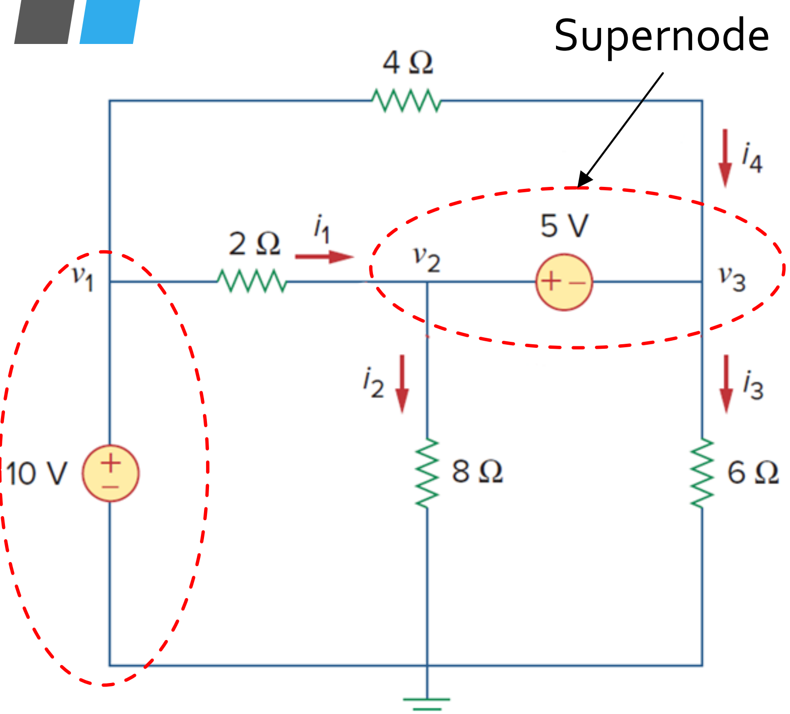
ที่โหนด 3 : $i_1 = 4i_x + i_3 ; \frac{V_1 - V_3}{2} = 4 * \frac{V_2}{4} + \frac{V_3}{6} \Rightarrow 3V_1 - 6V_2 - 4V_3 = 0$

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & 5 & 0 \\ 3 & -6 & -4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 = 32\text{V},\ v_2 = -25.6\text{V},\ v_3 = 62.4\text{V}$

# 3.3 Nodal Analysis with Voltage Sources (1)

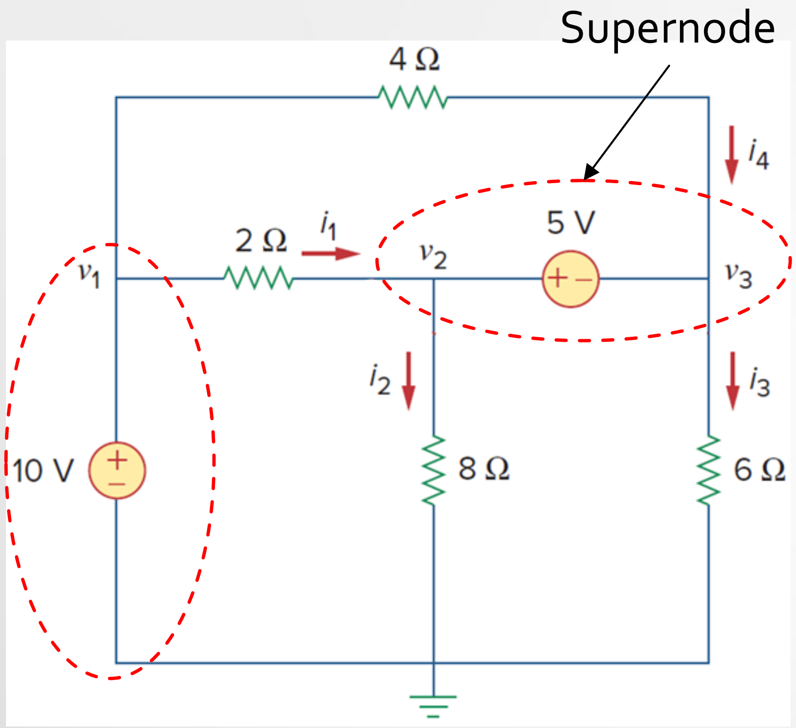
How voltage sources affect nodal analysis?




- Consider the following two possibilities.

- **CASE 1:** If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. for example, $v_1 = 10$ V
- **CASE 2:** If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages. In Fig. , nodes 2 and 3 form a supernode.

# 3.3 Nodal Analysis with Voltage Sources (1)

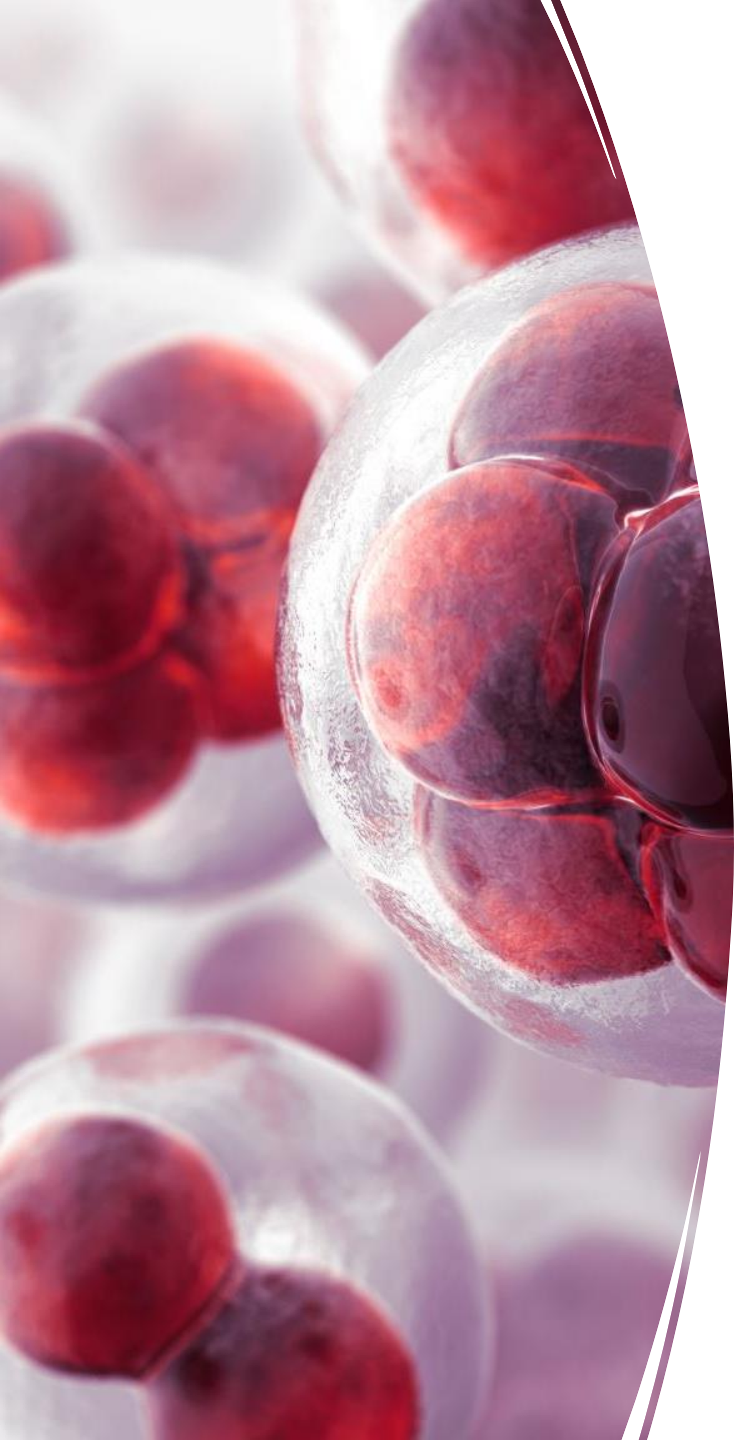



ที่ Supernode : $i_1 + i_4 = i_2 + i_3$ ; $\frac{10-V_2}{2} + \frac{10-V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6} \Rightarrow 36 = 3V_2 + 2V_3$

ที่ Supernode : $V_2 - V_3 = 5$ ;

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 5 \end{bmatrix}$$

$v_1 = 10\text{V},\ v_2 = 9.2\text{V},\ v_3 = 4.2\text{V}$



# 3.3 Nodal Analysis with Voltage Sources (3)

- A supernode is formed by <u>enclosing</u> a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it. (Supernode คือ node ที่ล้อมรอบ voltage source ซึ่งเชื่อมระหว่าง node ทั้งสอง)

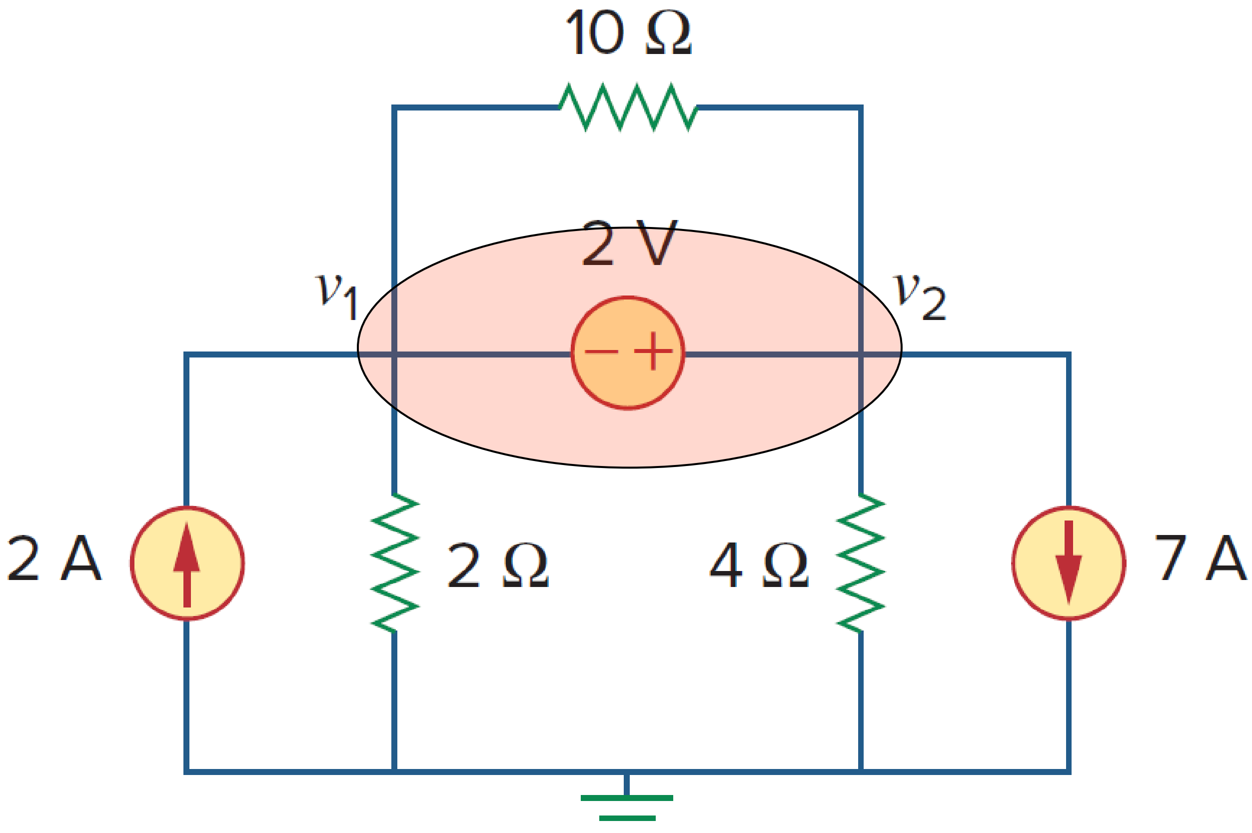
- <u>Note</u>: We analyze a circuit with the supernode using the same three steps mentioned above except that the supernode are treated differently.

# 3.3 Nodal Analysis with Voltage Sources (4)

**Note:** Properties of a supernode.

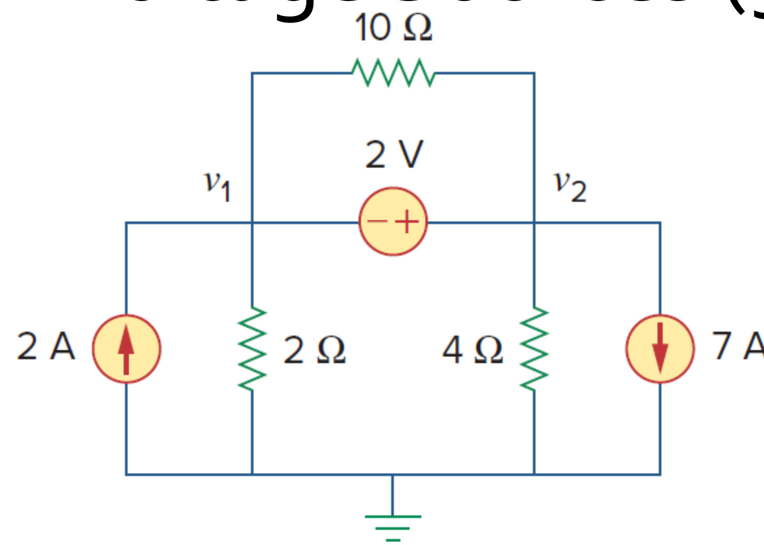
- The voltage source *inside the supernode* provides a constraint equation needed to solve for the node voltages.
- A supernode has *no voltage of its own*.
- A supernode requires the application of both KCL and KVL.

# 3.3 Nodal Analysis with Voltage Sources (2)



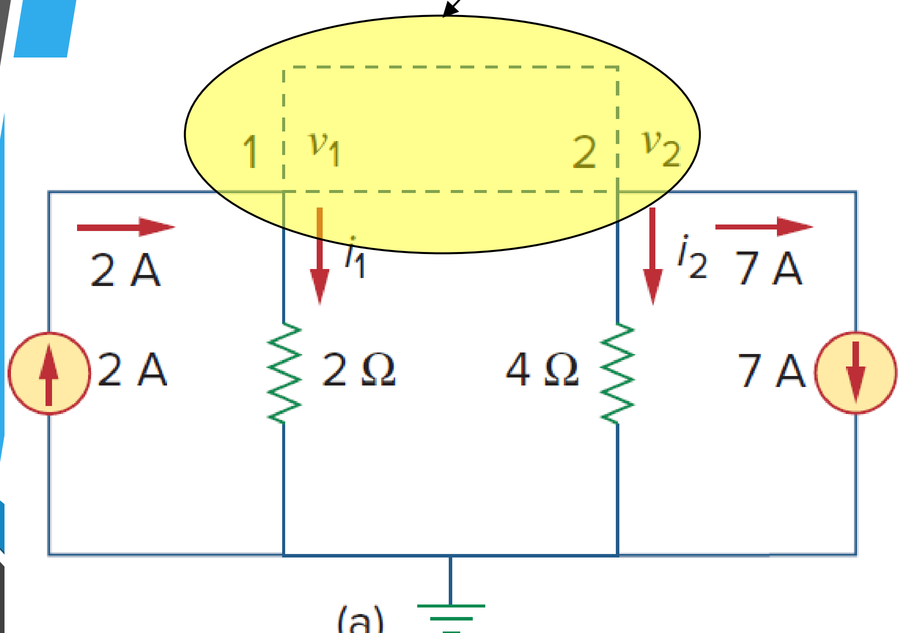

How to handle the 2V voltage source with Nodal Analysis?

# 3.3 Nodal Analysis with Voltage Sources (5)

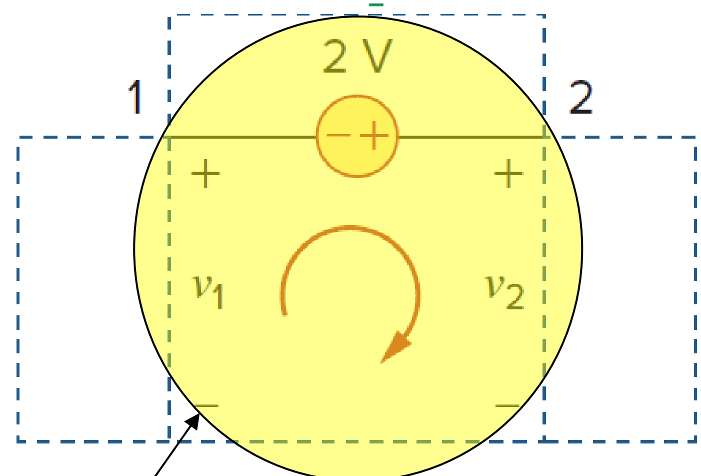



Ex.5 Find the node voltages.

Super-node => $2 - i_1 - i_2 - 7 = 0$




(a)



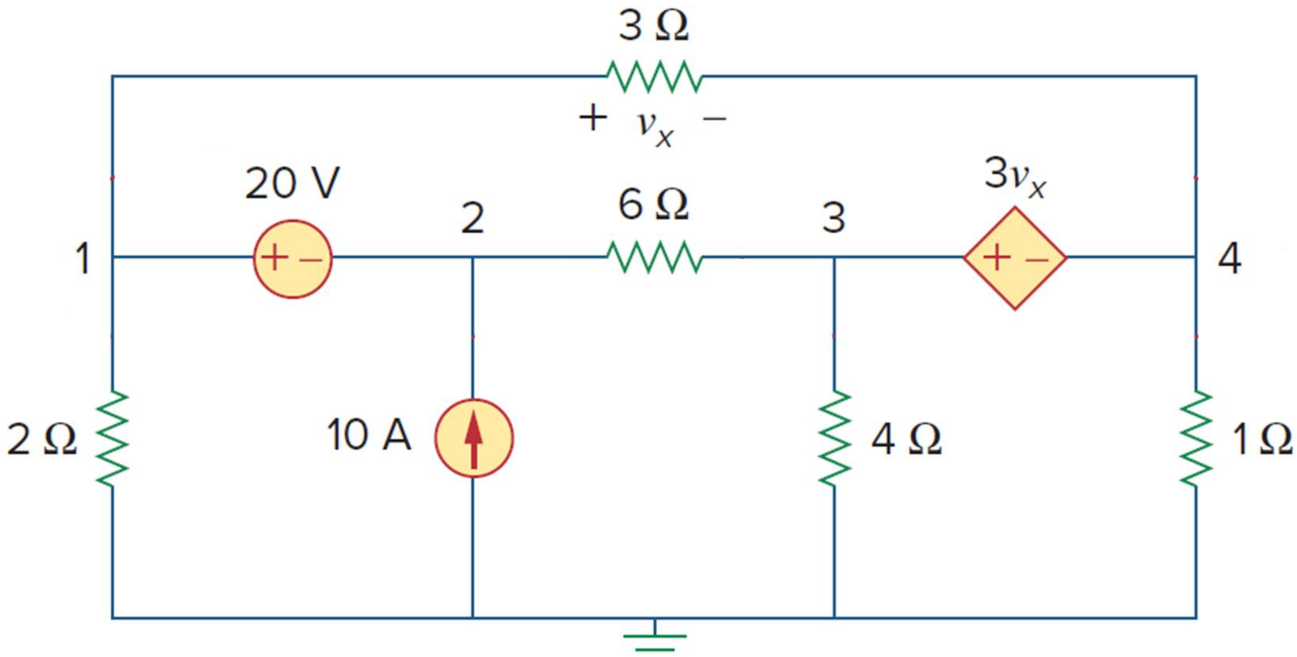

(b)

Apply KVL => $v_2 - v_1 - 2 = 0$

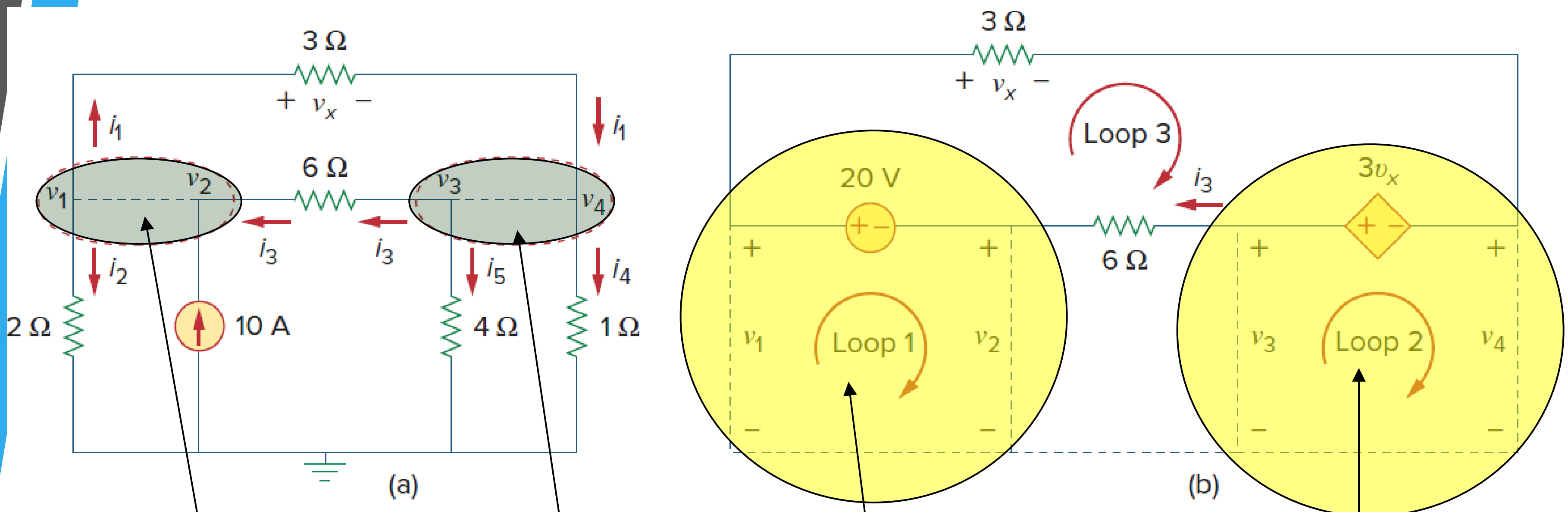
$v_1 = -7.333\text{V}$, $v_2 = -5.333\text{V}$

# 3.3 Nodal Analysis with Voltage Sources (6)

Ex.6 Find the node voltages in the circuit of Fig.



# 3.3 Nodal Analysis with Voltage Sources (7)




$$-i_1 - i_2 + i_3 + 10 = 0$$

$$-i_3 - i_5 - i_4 + i_1 = 0$$

$$v_1 - 20 - v_2 = 0$$

$$v_3 - 3v_x - v_4 = 0$$

$v_1 = 26.67\text{V}$, $v_2 = 6.667\text{V}$, $v_3 = 173.33\text{V}$, $v_4 = -46.67\text{V}$

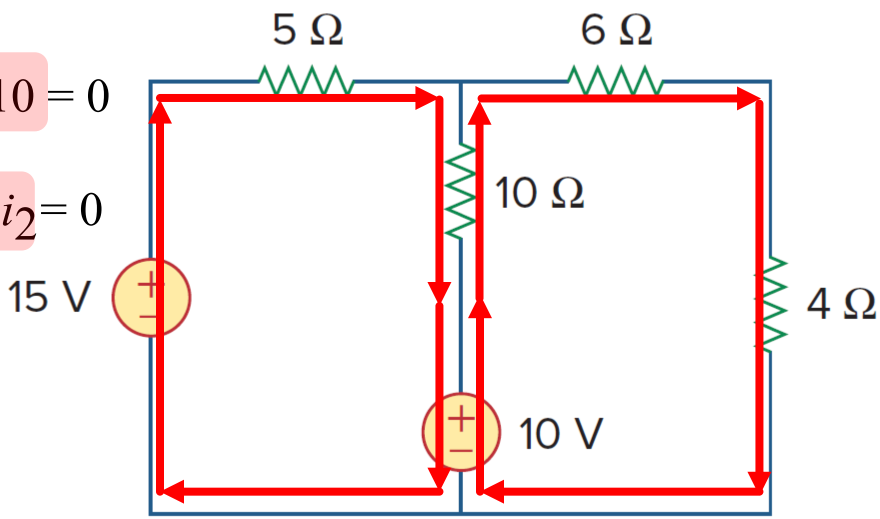
# 3.4 Mesh Analysis (1)

Steps:

1. Assign mesh currents $i_1$, $i_2$,… to the meshes.
2. Apply KVL to each of the meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting equations to get the mesh currents.

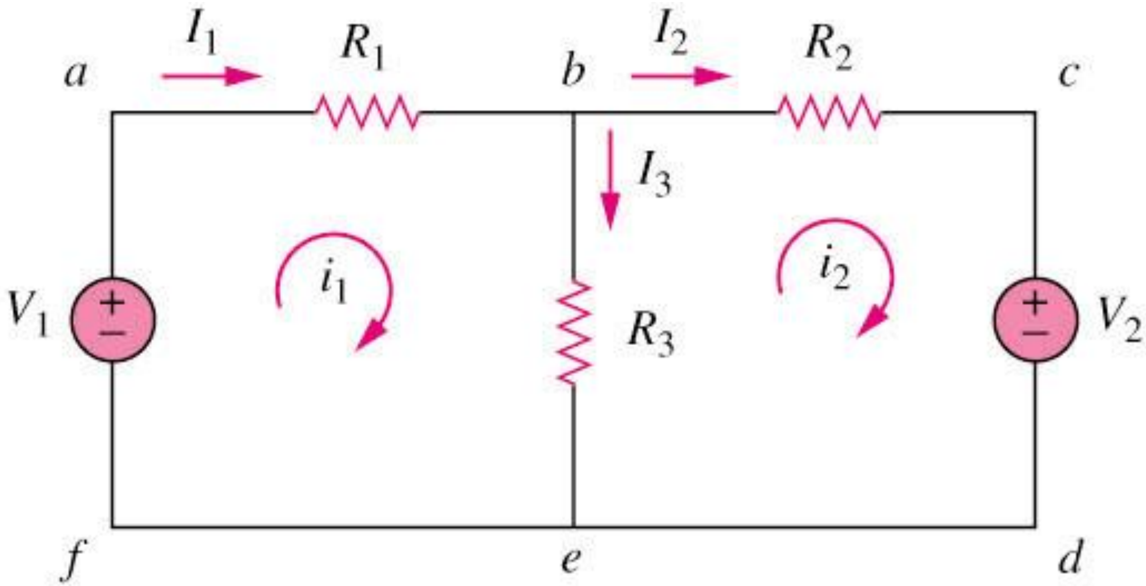
Apply KVL at mesh 1 : $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

Apply KVL at mesh 2 : $-10 + 10(i_2 - i_1) + 6i_2 + 4i_2 = 0$



# 3.4 Mesh Analysis (2)

Ex.7 Circuit with independent voltage sources




Note:

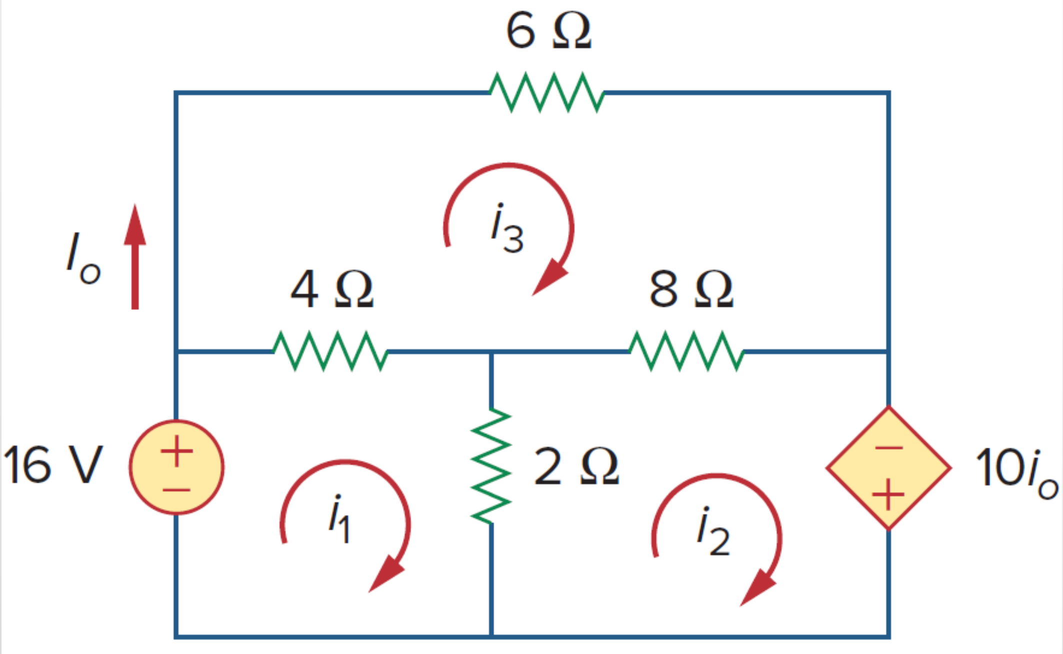
$i_1$ and $i_2$ are mesh current (imaginative, not measurable directly)

$I_1$, $I_2$ and $I_3$ are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

# 3.4 Mesh Analysis (3)

<u>Ex.8</u> Use mesh analysis to determine $I_o$




$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 10 & -18 \\ -4 & -8 & 18 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$$



$i_1$ = -2.5714 A, $i_2$ = -7.7143 A, $i_3 = I_o$= -4.0 A

# 3.5 Mesh Analysis with Current Source (1)

## How current sources affect mesh analysis?

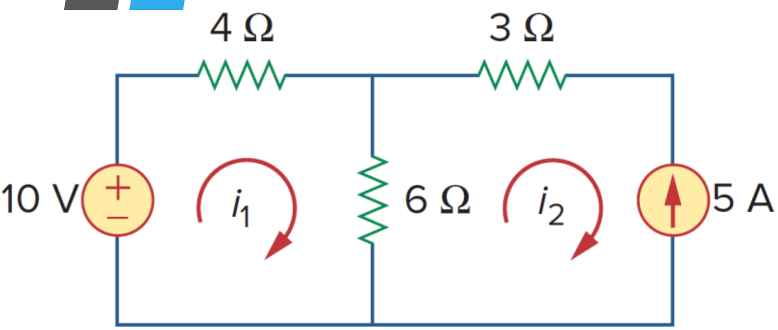



Fig.1

- Consider the following two possibilities.
- **CASE 1:** When a current source exists only in one mesh: In Fig.1, for example, we set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way; that is, $-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2$ A

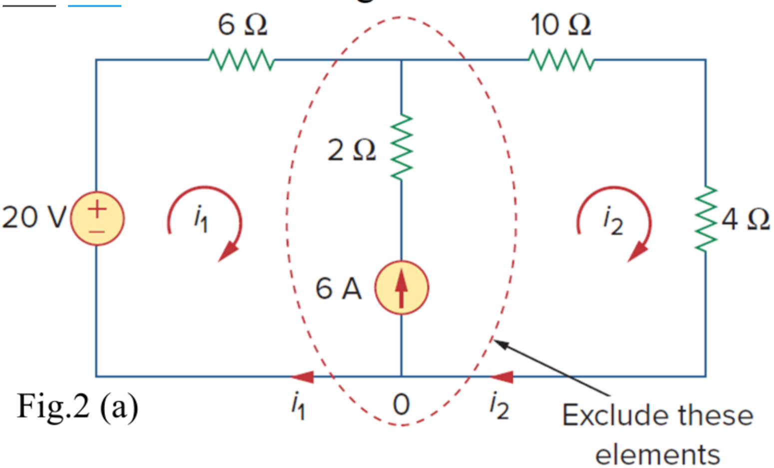



Fig.2 (a)

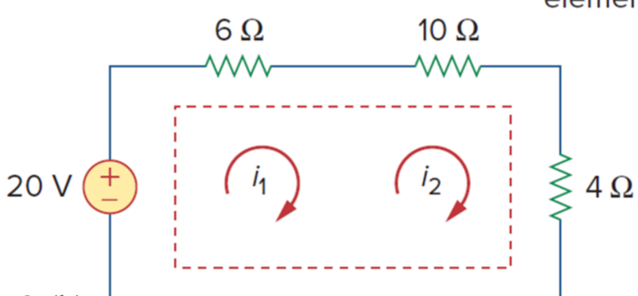
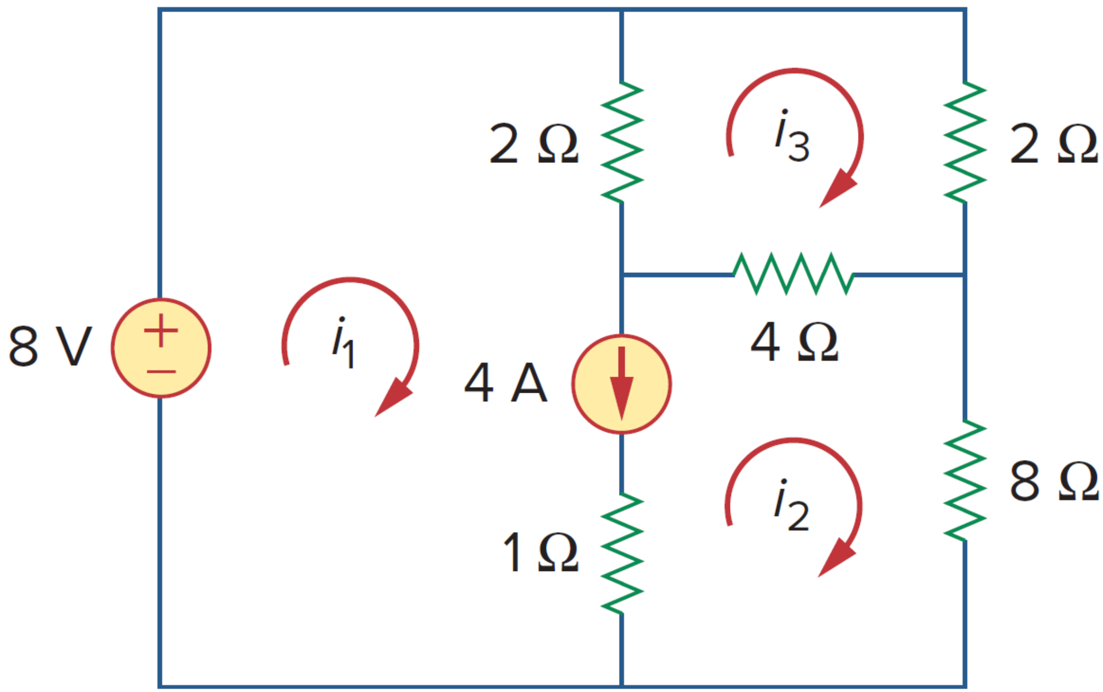



Fig.2 (b)

- **CASE 2:** When a current source exists between two meshes: Consider the circuit in Fig. (a), for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig. (b). Thus,
- $-20 + 6i_1 + 10i_2 + 4i_2 = 0 \Rightarrow 6i_1 + 14i_2 = 20$
- Applying KCL to node 0 in Fig. (a) gives
- $i_2 = i_1 + 6$
- Solving both Eqs. above/.
- $i_1 = -3.2$ A, $i_2 = 2.8$ A

# 3.5 Mesh Analysis with Current Source (2)

Ex.8 Use mesh analysis to determine $i_1$, $i_2$, and $i_3$




$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & -4 & 8 \\ 2 & 12 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$$

$i_1$ = 4.632 A, $i_2$ = 631.6 mA, $i_3$ = 1.4736 A.

# 3.5 Mesh Analysis with Current Source (3)

<u>The properties of a supermesh:</u>

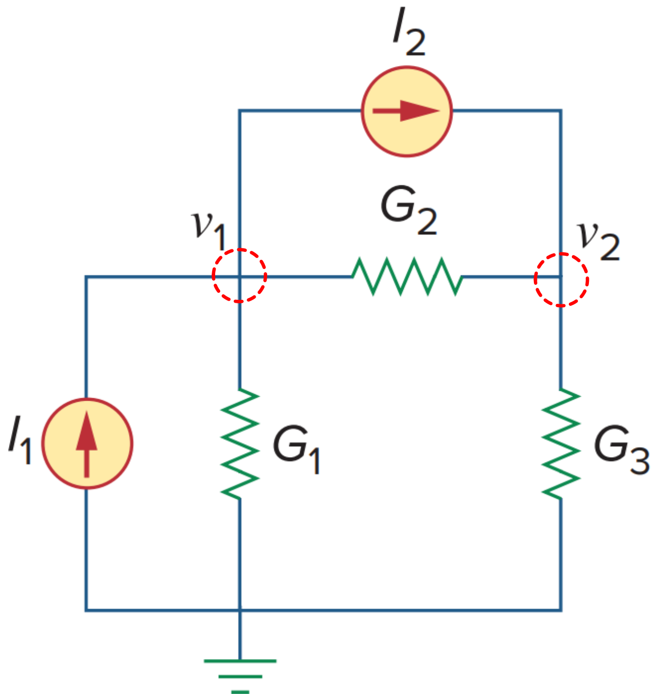
- The current source *<u>in the supermesh</u>* provides the constraint equation.
- A supermesh has *<u>no current of its</u>* own.
- A supermesh requires the application of both KVL and KCL.

# 3.6 Nodal Analysis by Inspection (1)

Each of the diagonal terms is the sum of the conductance connected directly to node 1 or 2,

while the off-diagonal terms are the negatives of the conductance connected between the nodes.

Each term on the right-hand side is the algebraic sum of the currents entering the node.



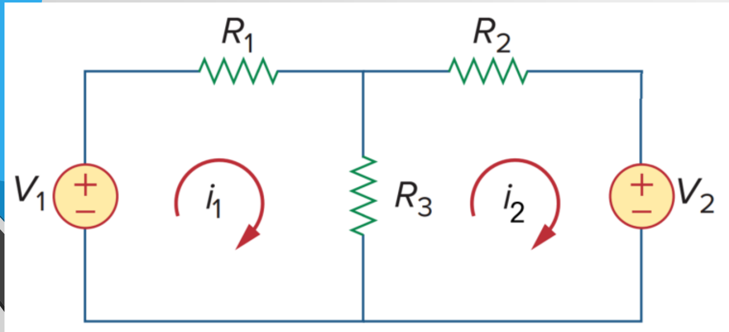

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

# 3.6 Mesh Analysis by Inspection (2)

Each of the diagonal terms is the sum of the resistances in the related mesh,

while each of the off-diagonal terms is the negative of the resistance common to meshes 1 and 2.

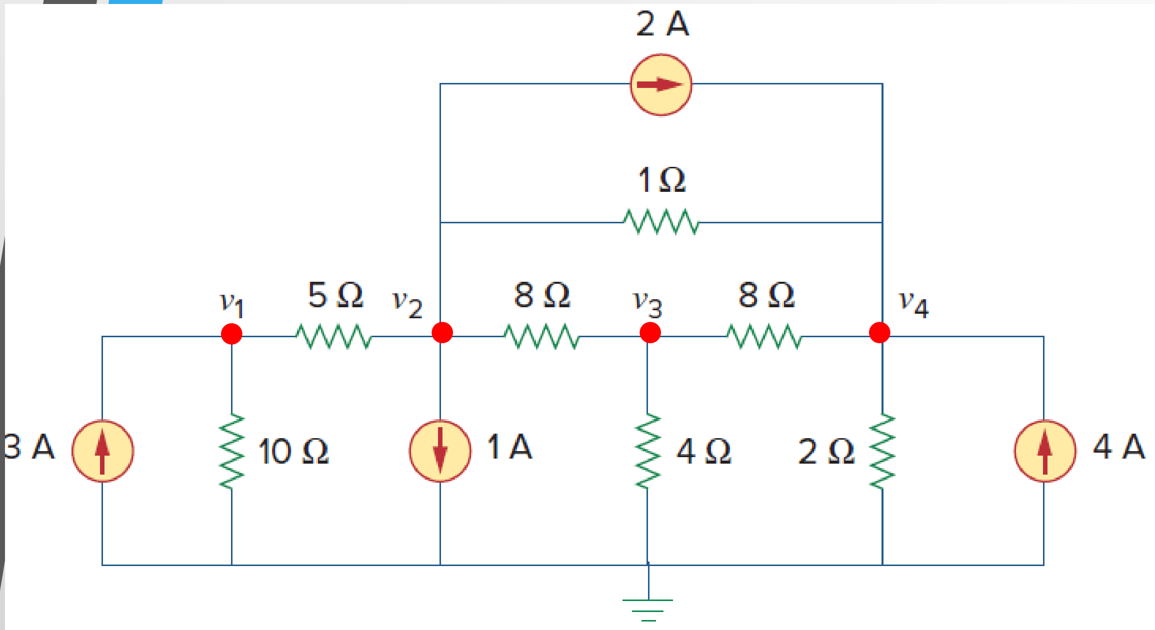
Each term on the right-hand side of Eq. is the algebraic sum taken clockwise of all independent voltage sources in the related mesh.




$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

# 3.6 Node Analysis by Inspection (3)

Ex.9 Write the nodal voltage equations for the circuit




$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \qquad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \qquad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \quad G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \qquad G_{32} = -0.125, \qquad G_{34} = -\frac{1}{8} = -0.125$$

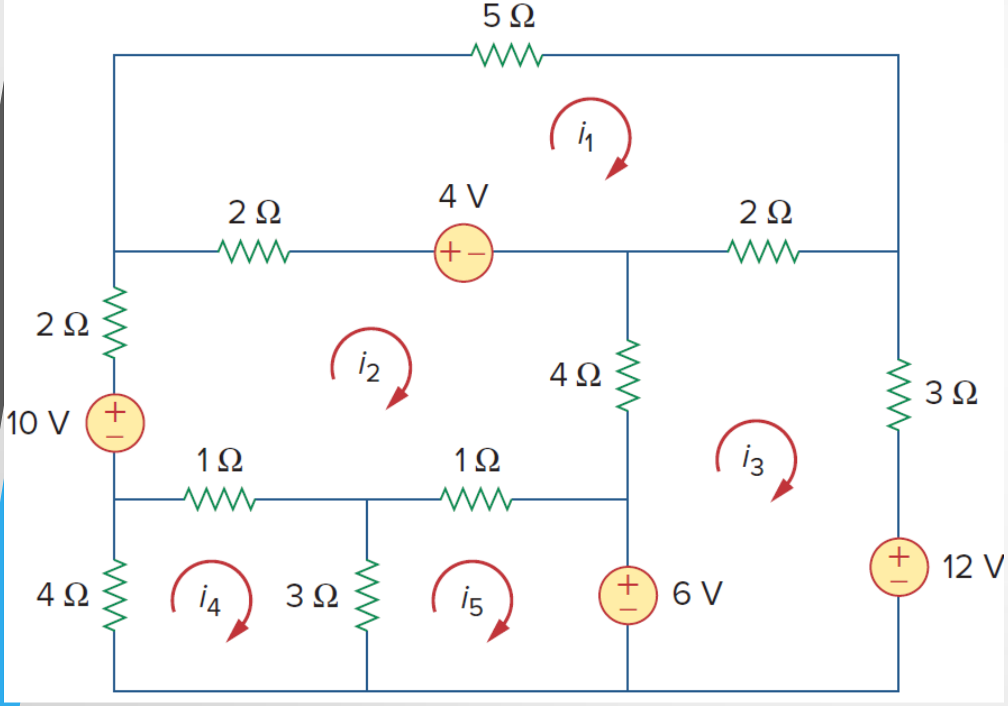
$$G_{41} = 0, \qquad G_{42} = -1, \qquad G_{43} = -0.125$$

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

$v_1$ = 13.897 V, $v_2$ = 5.845 V, $v_3$ = 3.348 V , $v_4$ = 7.547 V.

# 3.6 Mesh Analysis by Inspection (4)

Ex.10 Write the mesh-current equations for the circuit

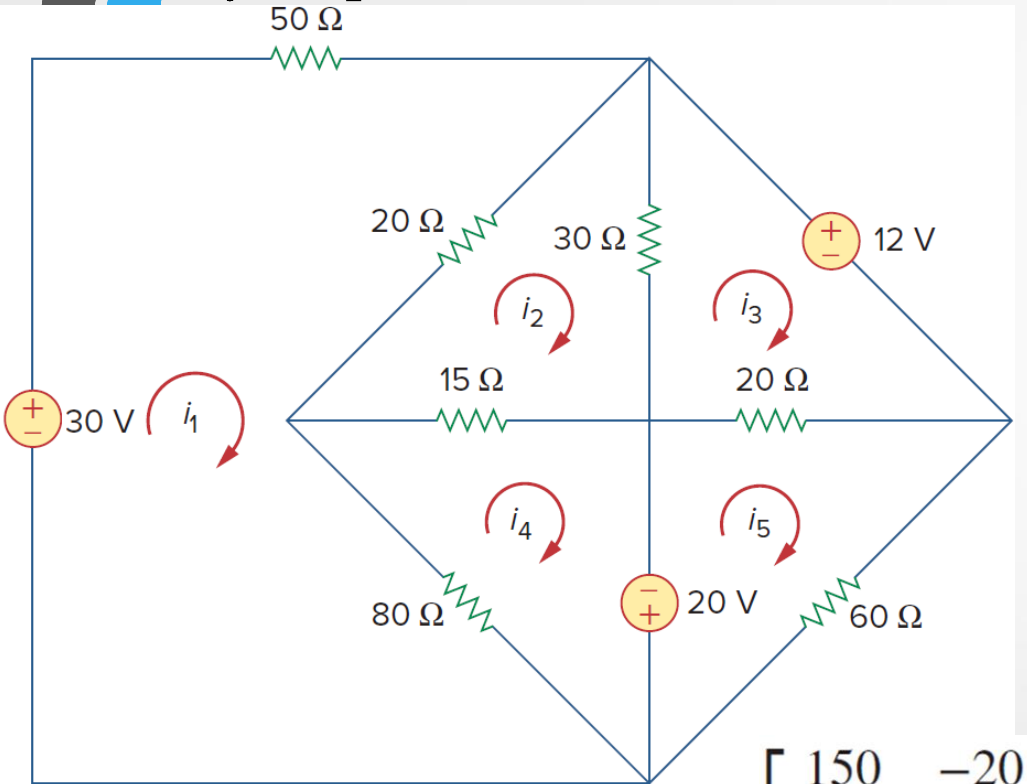



$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

$i_1$ = 0.3829 A, $i_2$ = 0.2110 A, $i_3$ = -0.4878 A, $i_4$ = -0.7184 A, $i_5$ = -1.986 A.

# 3.6 Mesh Analysis by Inspection (5)

Ex.11 By inspection, obtain the mesh-current equations for the circuit




$$\begin{bmatrix} 150 & -20 & 0 & -80 & 0 \\ -20 & 65 & -30 & -15 & 0 \\ -0 & -30 & 50 & 0 & -20 \\ -80 & -15 & 0 & 95 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix}$$



$i_1$ = 0.7039 A, $i_2$ = 0.3470 A, $i_3$ = -0.1464 A, $i_4$ = 0.8581 A, $i_5$ = -0.2866 A.

# 3.7 Nodal vs Mesh Analysis

➢ Which method is better or more efficient?

1. *Choose *nodal analysis* for circuit with fewer nodes than meshes.

   *Choose *mesh analysis* for circuit with fewer meshes than nodes.

   *Networks with parallel-connected elements, current sources, or super-nodes are more suitable for *nodal analysis*.

   *Networks that contain many series connected elements, voltage sources, or super-meshes are more suitable for *mesh analysis.*

2. If node voltages are required, applying *nodal analysis*.

   If branch or mesh currents are required, applying *mesh analysis.*

Be familiar with both methods of analysis