Electrical Engineering 1
12026105

Chapter 6

Capacitors and Inductors

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Fully understand the volt-amp characteristics of capacitors and inductors and their use in basic circuits.
2. Explain how capacitors behave when combined in parallel and in series.
3. Understand how inductors behave when combined in parallel and in series.
4. Know how to create integrators using capacitors and op amps.
5. Learn how to create differentiators and their limitations.
6. Learn how to create analog computers and to understand how they can be used to solve linear differential equations.

วัตถุประสงค์การเรียนรู้

โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:

1. เข้าใจคุณลักษณะของโวลต์-แอมป์ของตัวเก็บประจุและขดลวดเหนี่ยวนำและการใช้งานในวงจรไฟฟ้าพื้นฐาน
2. เข้าใจตัวเก็บประจุทำงานอย่างไรเมื่อต่อแบบขนานและอนุกรม
3. เข้าใจขดลวดเหนี่ยวนำทำงานอย่างไรเมื่อต่อแบบขนานและอนุกรม
4. รู้วิธีสร้างวงจรอินทิเกรเตอร์โดยใช้ตัวเก็บประจุและออปแอมป์
5. รู้วิธีสร้างวงจรดิฟเฟอร์เรนชิเอเตอร์และข้อจำกัดของมัน
6. เรียนรู้วิธีสร้างคอมพิวเตอร์แอนาล็อกและทำความเข้าใจว่าสามารถใช้แก้สมการอนุพันธ์เชิงเส้นได้

Capacitors and Inductors Chapter 6

6.1 Introduction

6.2 Capacitors (ตัวเก็บประจุ)

6.3 Series and Parallel Capacitors (ต่ออนุกรม,ขนานตัวเก็บประจุ)

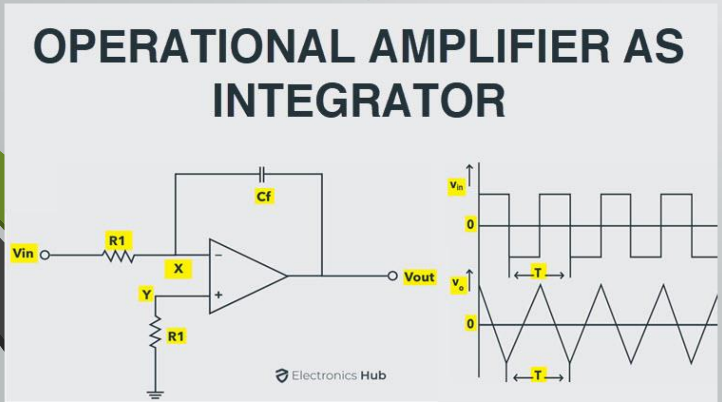
6.4 Inductors (ขดลวดเหนี่ยวนำ)

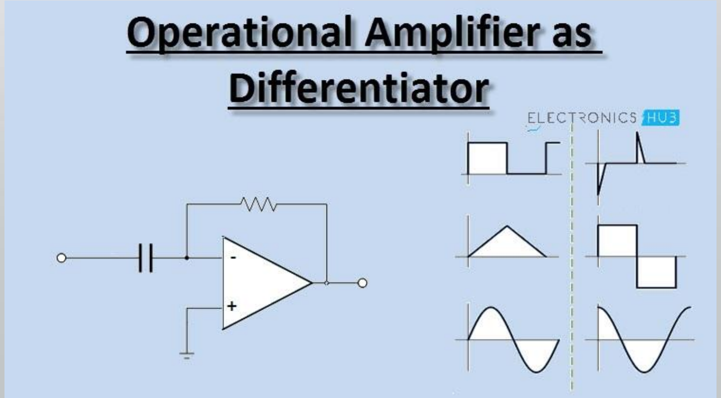
6.5 Series and Parallel Inductors (ต่ออนุกรม,ขนานขดลวดเหนี่ยวนำ)

6.6 Applications

6.1 Introduction

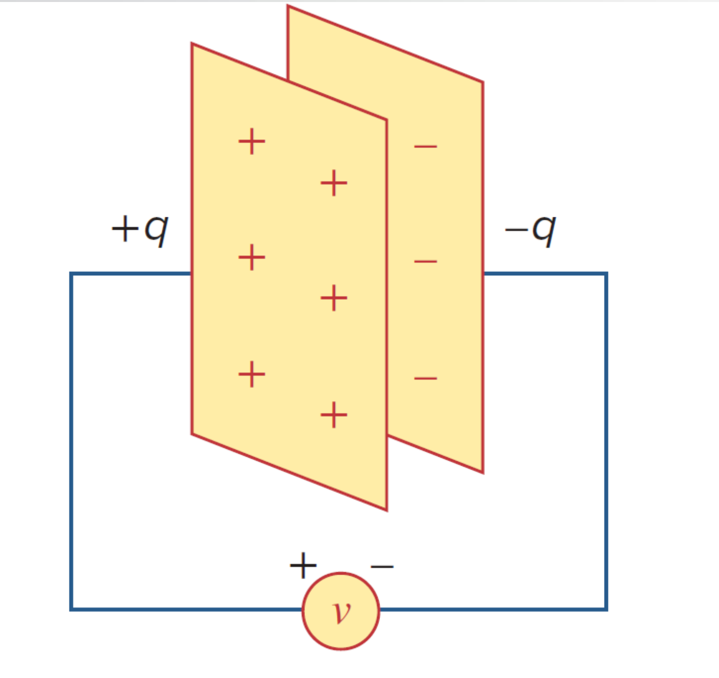
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved later.
- For this reason, capacitors and inductors are called storage elements.
- As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.



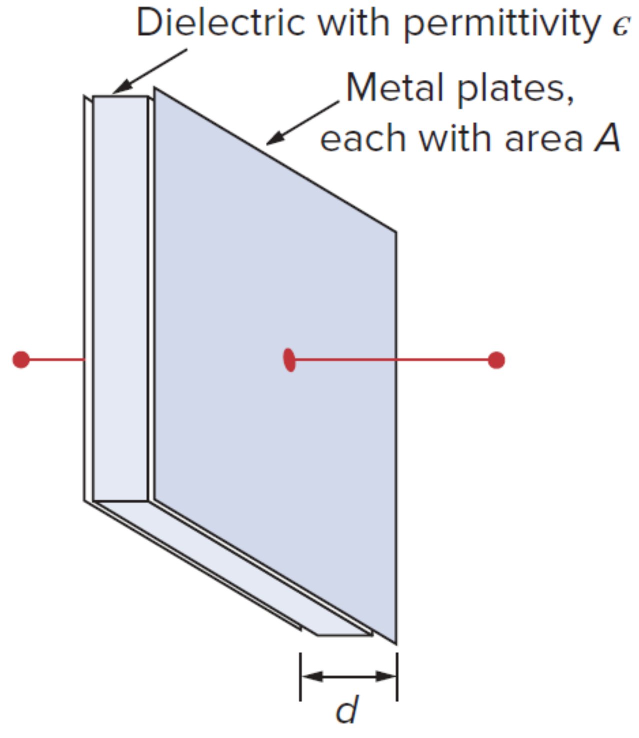


6.2 Capacitors (1)

- A capacitor is a passive element designed to **store energy** in its **electric field.**



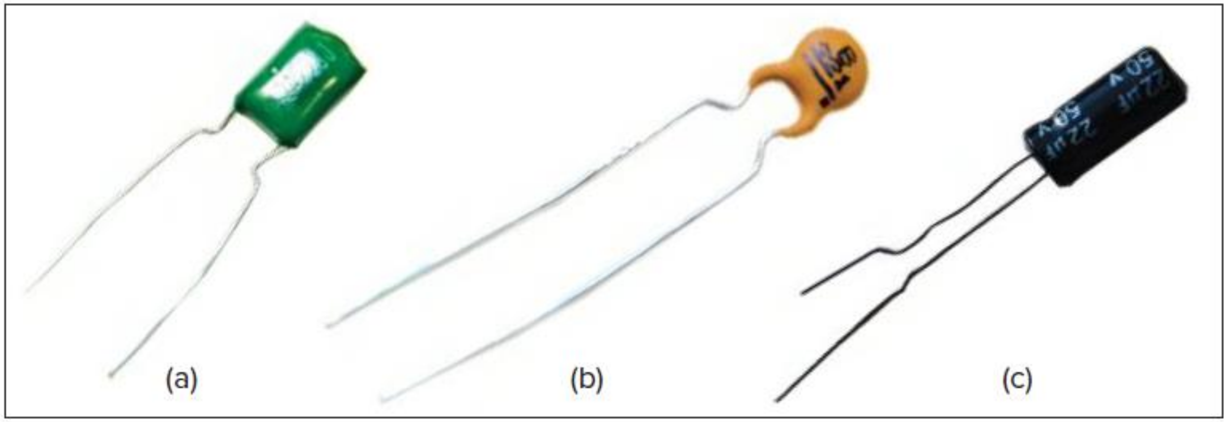
A capacitor with applied voltage *v*.



A typical capacitor.

- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.2 Capacitors (2)

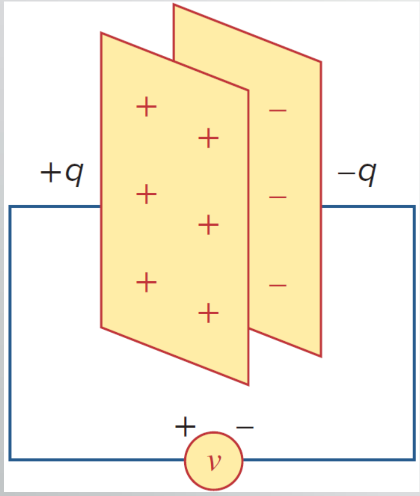


Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
Mark Dierker/McGraw-Hill Education

Variable capacitor.

6.2 Capacitors (3)

- **Capacitance** C is the ratio of the charge q on one plate of a capacitor to the voltage difference V between the two plates, measured in farads (F).

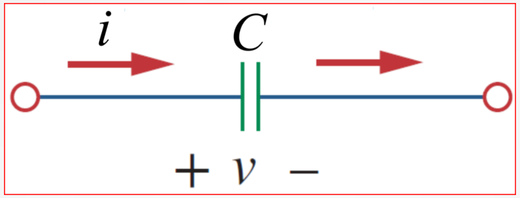


$$i = C\frac{dv}{dt}$$ and $$C = \frac{q}{V} = \frac{\varepsilon A}{d}$$

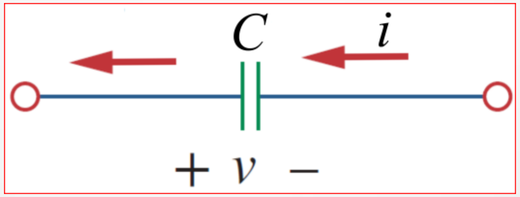
- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates. ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m)
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

6.2 Capacitors (4)

- If *i* is flowing into the positive (+) terminal of *C*
  - Charging => *i* is positive (+)



  - Discharging => *i* is negative (-)



- The *i*-*v* relationship of capacitor according to above convention is

$$i = C\frac{dv}{dt}$$

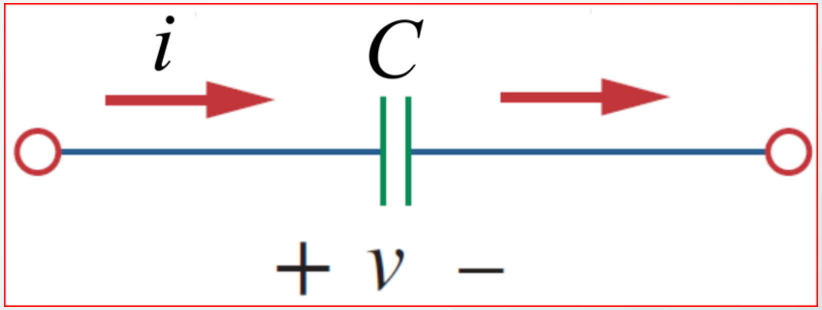
and

$$v = \frac{1}{C}\int_{t_0}^{t} i\,d\,t + v(t_0)$$

6.2 Capacitors (5)

- The energy, ***w***, stored in the capacitor is

$$w = \frac{1}{2} C v^2$$



- A capacitor is
  - an **open circuit** to dc (กระแสตรง)
  - its voltage **cannot change abruptly**.

$$(i = C\frac{dv}{dt} = 0)$$

6.2 Capacitors (6)

Ex.1 The current through a 100-μF capacitor is $i(t)$ = 50sin(120πt) mA. Calculate the voltage across it at t =1 ms and t = 5 ms. Take $v(t_0) = v(0) = 0$.

$$v = \frac{1}{C}\int_{t_0}^{t} i\, d\, t + v(t_0)$$

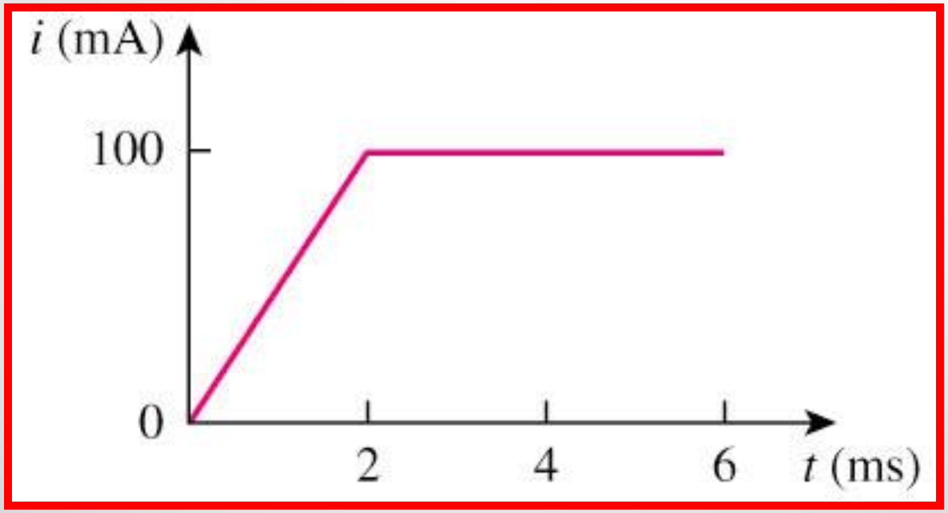
$$v = \frac{50mA}{100\mu F}\int_0^{1ms} \sin(120\pi t)\, dt = 500\left(-\frac{\cos(120\pi t)}{120\pi}\bigg|_0^{1ms}\right) = 1.3263(1 - \cos(0.377))$$
$$= 93.14mV$$

$$v = \frac{50mA}{100\mu F}\int_0^{5ms} \sin(120\pi t)\, dt = 500\left(-\frac{\cos(120\pi t)}{120\pi}\bigg|_0^{5ms}\right) = 1.3263(1 - \cos(1.885))$$
$$= 1.7361mV$$

Answer: v(1ms) = 93.14mV , v(5ms) = 1.7361V

6.2 Capacitors (7)

Ex.2 An initially uncharged 1mF capacitor has the current shown below across it. Calculate the voltage across it at t = 2 ms and t = 5 ms.



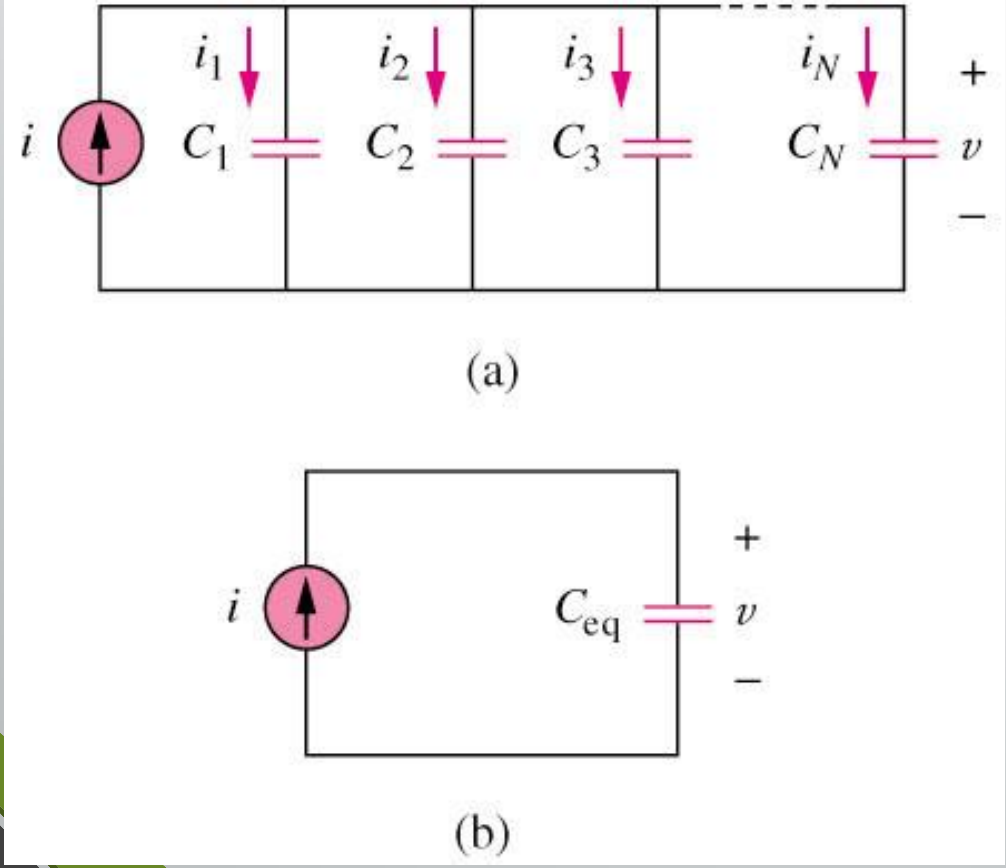
$$v = \frac{1}{1\text{mF}} \int_0^{2ms} 50t\,dt = 1000\left(25t^2\Big|_0^{2ms}\right) = 0.1V = 100mV$$

$$v = \frac{1}{1\text{mF}} \int_{2ms}^{5ms} 100m\,dt + 100mV = 100(3mV) + 100mV = 400mV$$

Answer: v(2ms) = 100 mV , v(5ms) = 400 mV

6.3 Series and Parallel Capacitors (1)

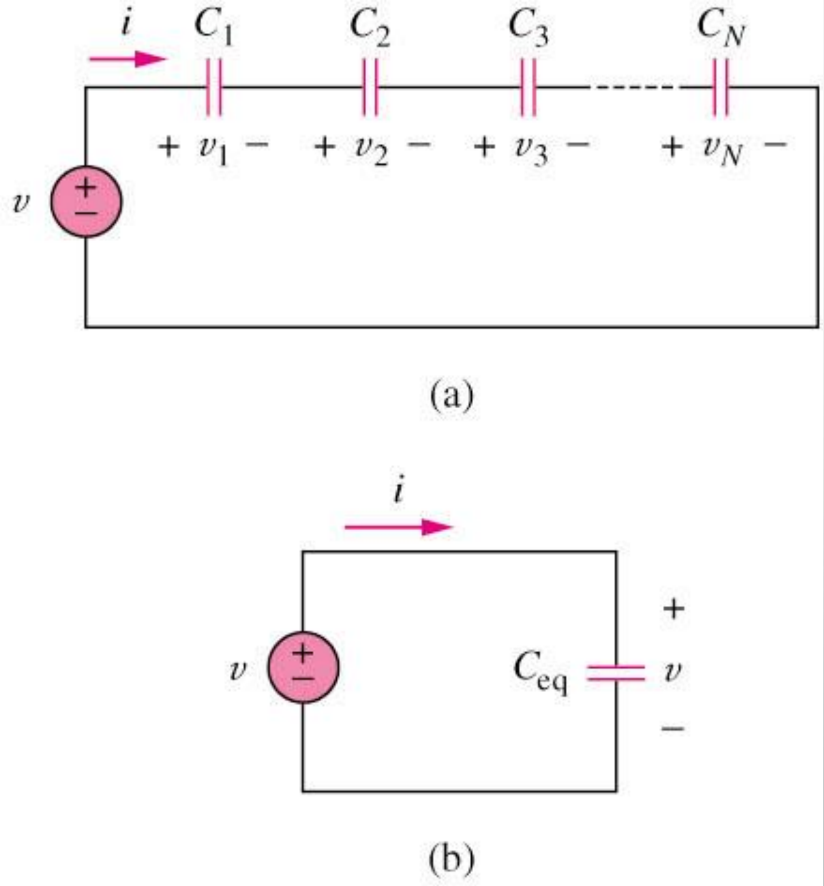
- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



$$C_{eq} = C_1 + C_2 + ... + C_N$$

6.3 Series and Parallel Capacitors (2)

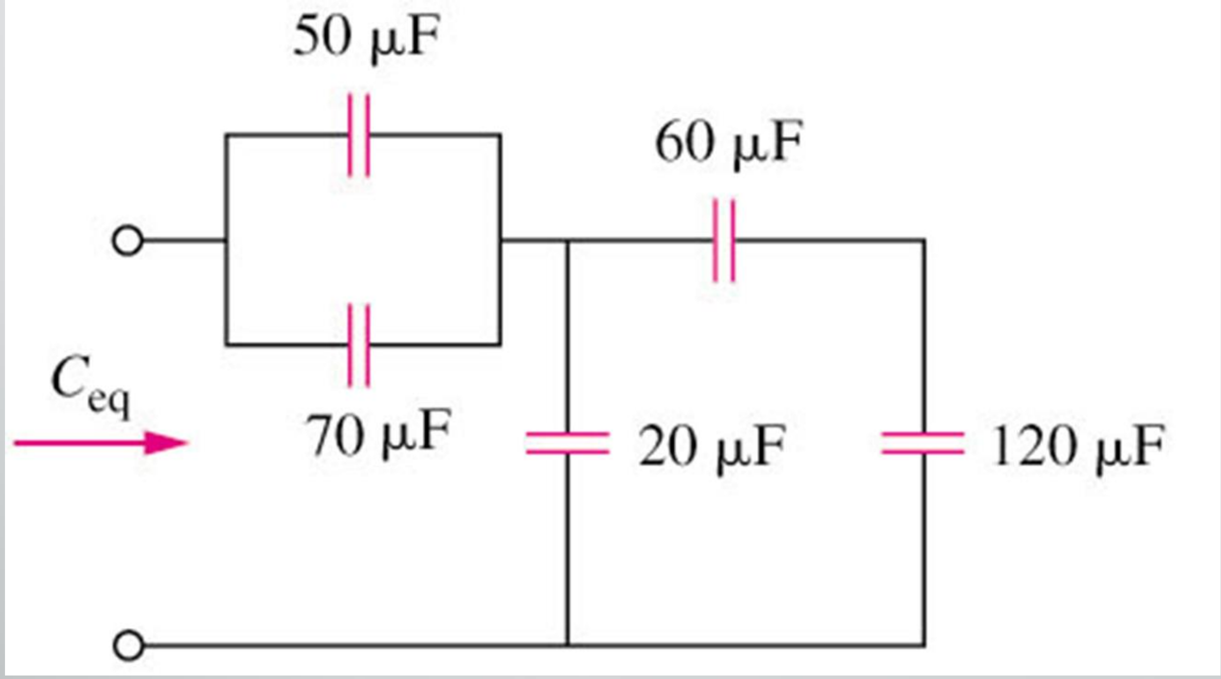
- The equivalent capacitance of N **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

6.3 Series and Parallel Capacitors (3)

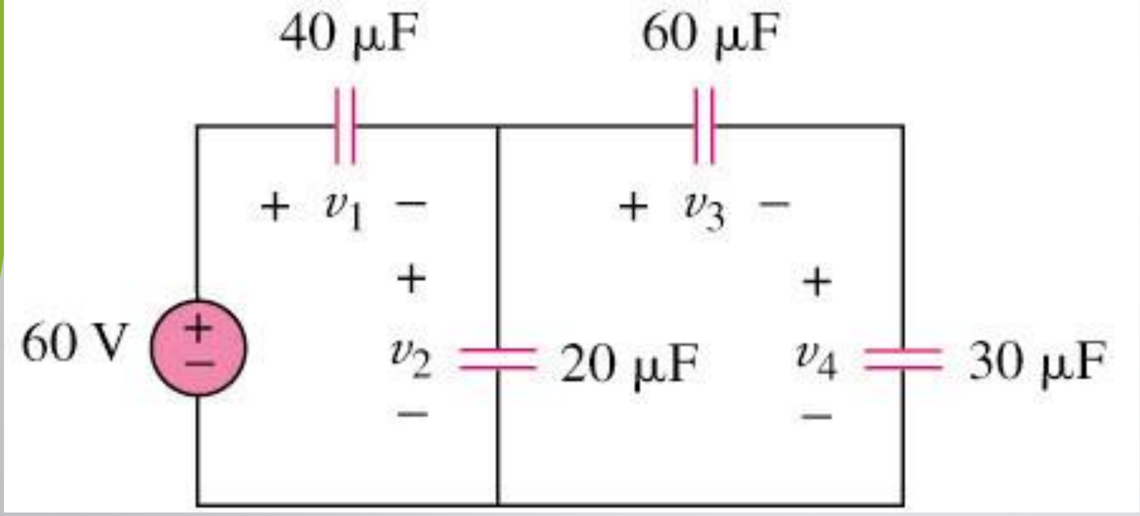
<u>Ex.3</u> Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:

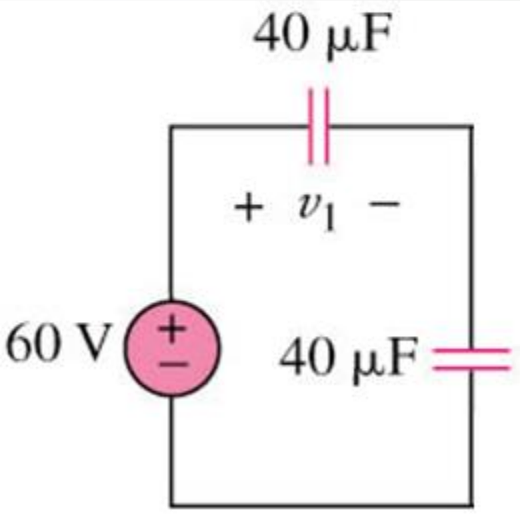


Answer: C_{eq} = 40μF

6.3 Series and Parallel Capacitors (4)

Ex.4 Find the voltage across each of the capacitors in the circuit shown below:

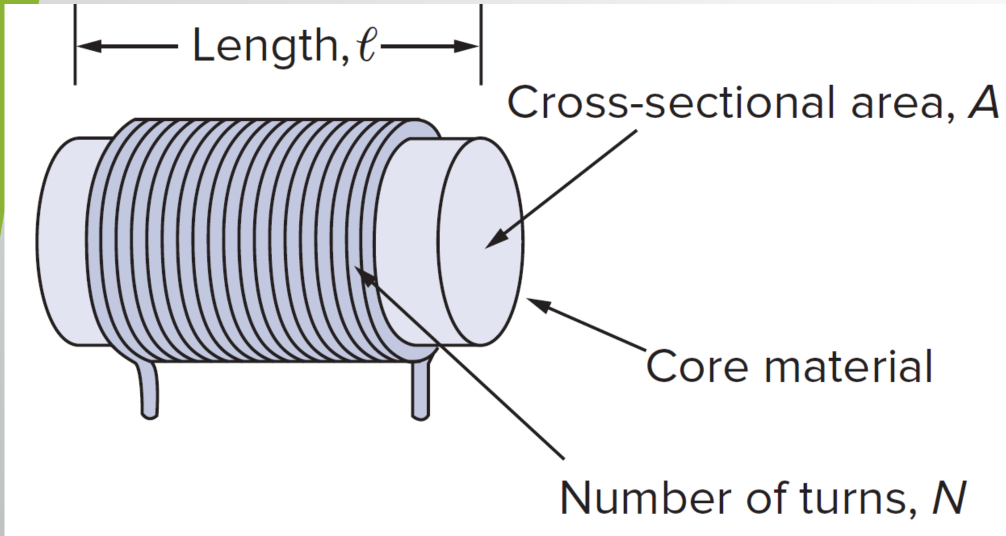




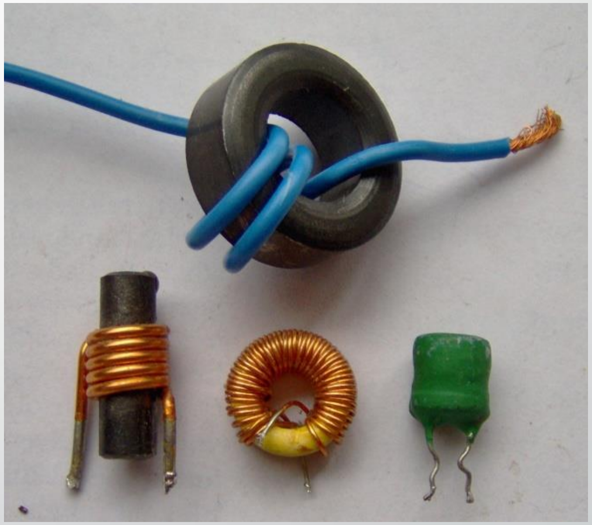
Answer: v_1 = 30V, v_2 = 30V, v_3 = 10V, v_4 = 20V

6.4 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



Typical form of an inductor.



- An inductor consists of a coil of conducting wire.

6.4 Inductors (2)

- Inductance is the property whereby an inductor exhibits
- opposition to the change of current flowing through it, measured in henrys (H). ($\mu_0 = 4\boldsymbol{\pi}\times 10^{-7}$ H/m)

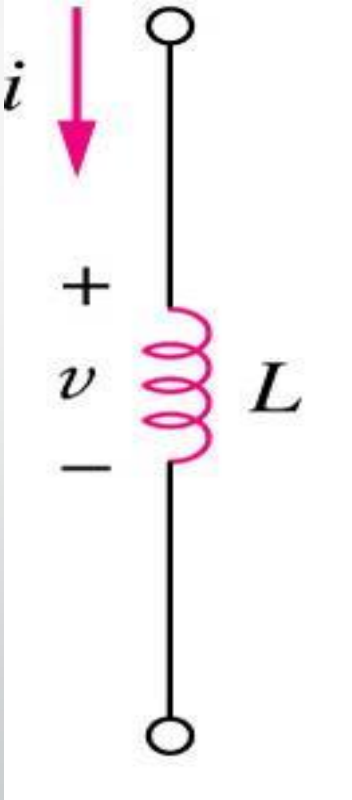
$$v = L\frac{di}{dt}$$ and $$L = \frac{\mu A N^2}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and nH (10^{-9}).

6.4 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



- The power stored by an inductor:

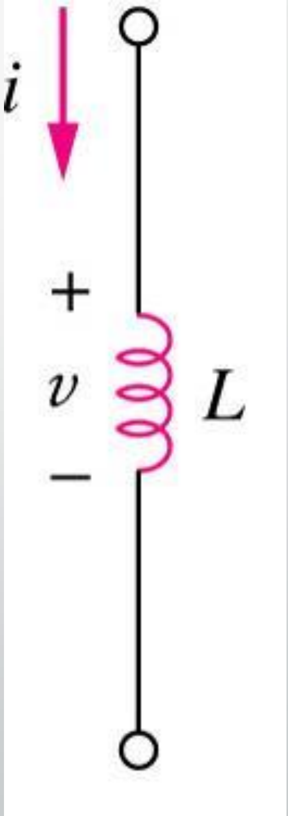
$$w = \frac{1}{2}Li^2$$

- An inductor acts like a short circuit to dc (di/dt = 0) and its current cannot change abruptly.

6.4 Inductors (4)

<u>Ex.5</u> The terminal voltage of a 2 H inductor is v = 10(1-t) V

Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s. Assume i(0) = 2 A.



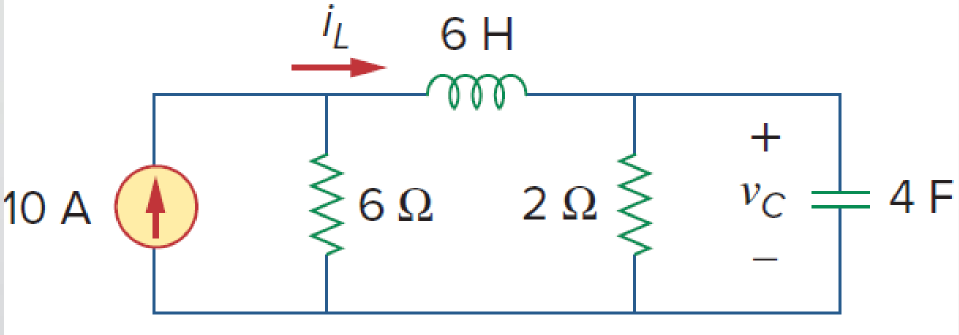
$$i = \frac{1}{2}\int_0^4 10(1\text{-}t)\ dt + 2 = \frac{1}{2}\left(10t - 5t^2\Big|_0^4\right) + 2 = -20 + 2 = -18V$$

$$w = \frac{1}{2}Li^2 - \frac{1}{2}Li_o^2 = \frac{1}{2}(2)(\text{-}18)^2 - 4 = 324\text{J} - 4\text{J} = 320\text{J}$$

Answer: i(4s) = -18V , w(4s) = 320J

6.4 Inductors (5)

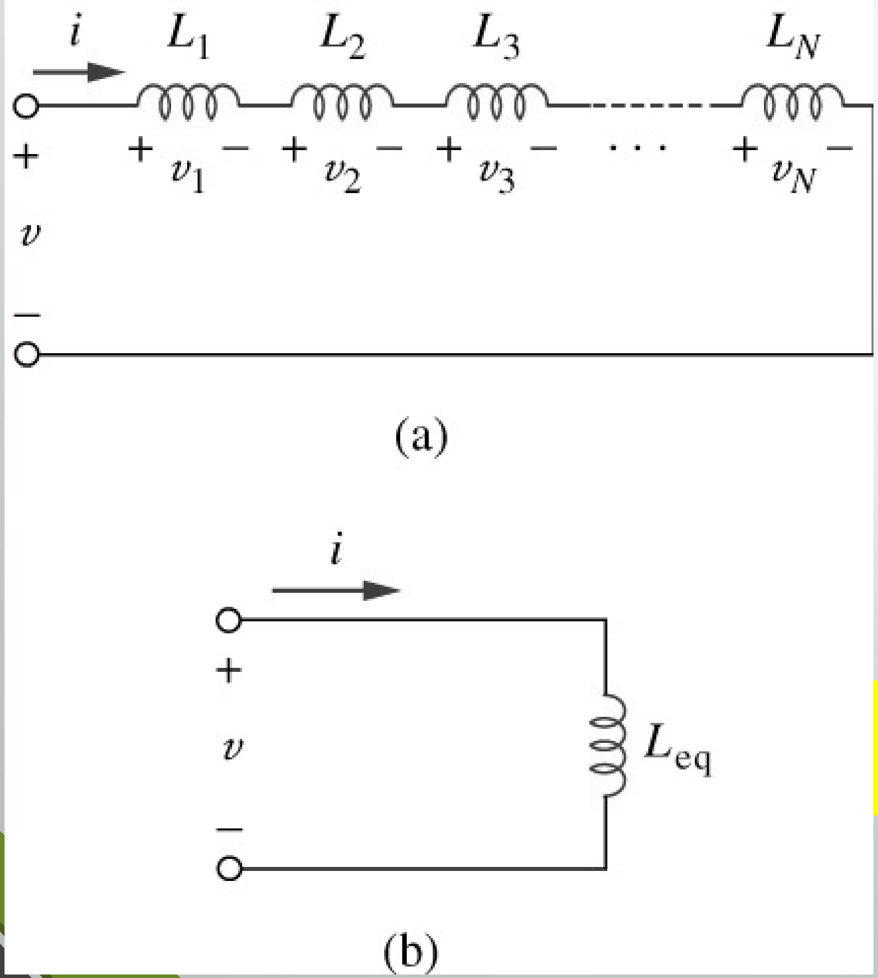
Ex.6 Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit shown below under dc conditions.



$i_L = 7.5A,\ v_C = 15V,\ w_C = 450\text{J},\ w_L = 168.75\text{J}$

6.5 Series and Parallel Inductors (1)

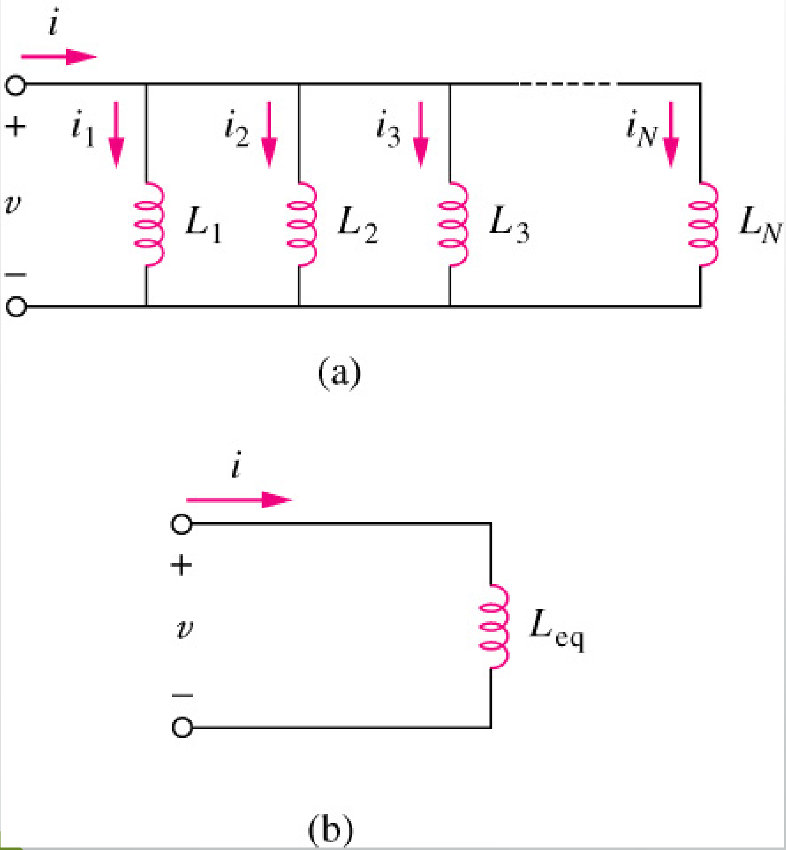
- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

6.5 Series and Parallel Inductors (2)

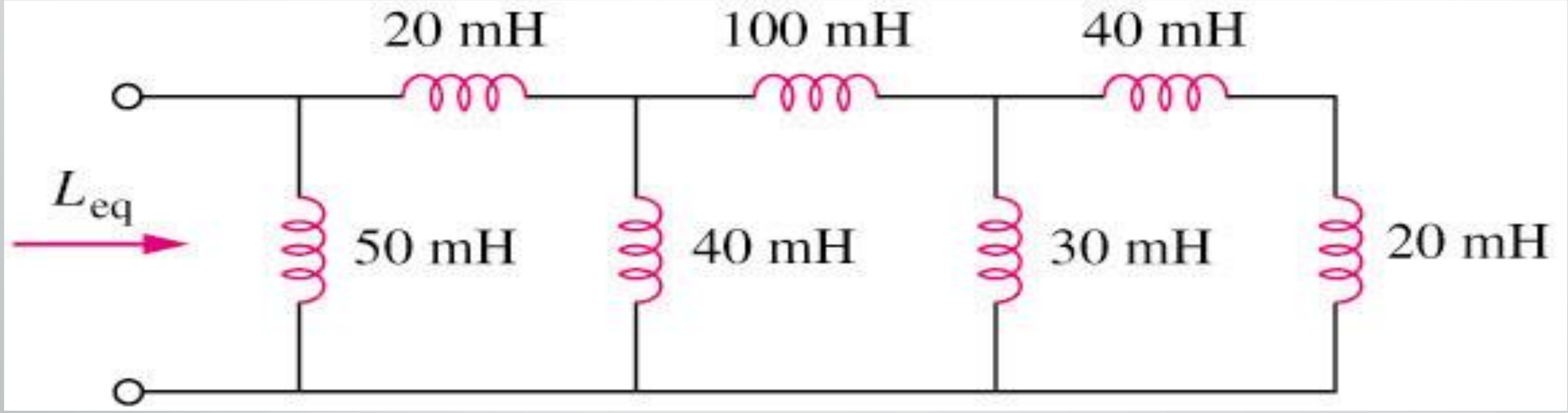
- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

6.5 Series and Parallel Capacitors (3)

Ex.7 Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:

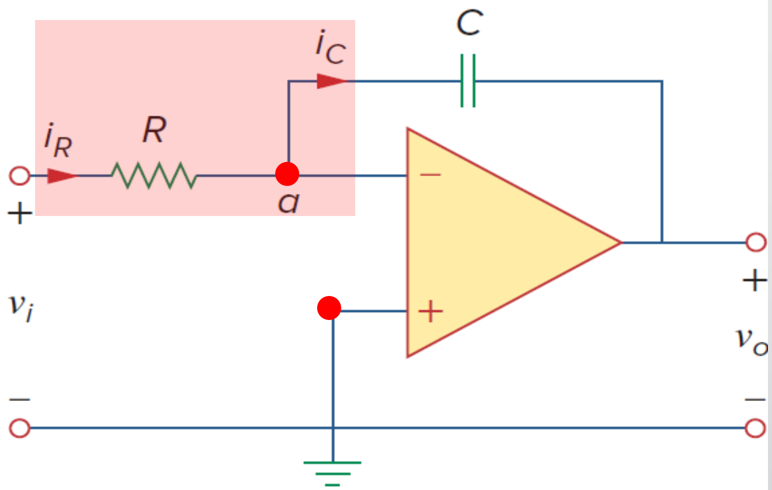


Answer: L_{eq} = 25mH

6.6 Applications (Integrator)

An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

It is interesting that we can obtain a mathematical representation of integration this way. At node *a* in Fig.



$$i_R = i_C$$

But

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

Substituting these in Eq., we obtain

$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$dv_o = -\frac{1}{RC}v_i\,dt$$

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

6.6 Applications (Differentiator)

A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

Applying KCL at node a,

$$i_R = i_C$$

But $$i_R = -\frac{v_o}{R}, \qquad i_C = C\frac{dv_i}{dt}$$

Substituting these in Eq. yields

$$v_o = -RC\frac{dv_i}{dt}$$

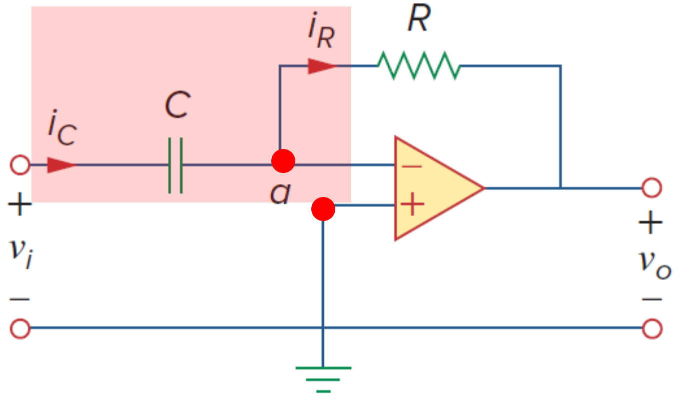


TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i