

# Electrical Engineering 1 12026105

## Chapter 12

## **Three-Phase Circuit**

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#### Learning Objectives

By using the information and exercises in this chapter you will be able to:

- 1. Understand balanced three-phase voltages.
- 2. Analyze balanced wye-wye circuits.
- 3. Understand and analyze balanced wye-delta circuits.
- 4. Analyze balanced delta-delta circuits.
- 5. Understand and analyze balanced delta-wye circuits.
- 6. Explain and analyze power in balanced three-phase circuits.
- 7. Analyze unbalanced three-phase circuits.

## วัตถุประสงค์การเรียนรู้

- โดยใช้ข้อมูลและแบบฝึกหัดในบทนี้ นักเรียนจะสามารถ:
- 1. เข้าใจสมดุลแรงดันไฟฟ้าสามเฟส
- วิเคราะห์วงจรสมดุล Y Y ได้
- เข้าใจและวิเคราะห์วงจรสมดุล Y ∆ ได้
- 4. วิเคราะห์วงจรสมดุล  $\Delta \Delta$  ได้
- เข้าใจและวิเคราะห์วงจรสมดุล △ Yได้
- 6. อธิบายและวิเคราะห์กำลังในวงจรสมดุลสามเฟสได้
- 7. วิเคราะห์วงจรสามเฟสที่ไม่สมดุลได้



## Three-Phase Circuits Chapter 12 What is a Three-Phase Circuit? Balance Three-Phase Voltages Balance Three-Phase Connection Power in a Balanced System Unbalanced Three-Phase Systems Application – Residential Wiring



Nikola Tesla (1856–1943)



## 12.1 What is a Three-Phase Circuit?

So far, we have dealt with single-phase circuits.



Single-phase systems: (a) two-wire type, (b) three-wire type.



## 12.1 What is a Three-Phase Circuit?

It is a system produced by a generator consisting of <u>three sources</u> having the same amplitude and frequency but <u>out of phase</u> with each other by 120°.





## 12.1 What is a Three-Phase Circuit? Advantages:

- 1. <u>Nearly all</u> electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz (or  $\omega = 377 \text{ rad/s}$ ) in the U.S. or 50 Hz (or  $\omega = 314 \text{ rad/s}$ ) in some other parts of the world.
- 2. The instantaneous power in a three-phase system can be <u>constant (not pulsating)</u>. This results in uniform power transmission and less vibration of three-phase machines.
- 3. For the same amount of power, the three-phase system is more <u>economical</u> that the single-phase. In fact, the amount of wire required for a three-phase system is <u>less</u> <u>than</u> that required for an equivalent single-phase system.



A three-phase ac generator (or alternator) consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).





- A typical three-phase system consists of three voltage sources connected to loads by three or four wires.
- The voltage sources can be either wye connected as shown in Fig. (a) or delta-connected as in Fig. (b).





Consider the wye-connected voltages. There are two possible combinations.  $v_{cn}$ 



 $\mathbf{V}_{an} = V_p / \underline{0^{\circ}}$  $\mathbf{V}_{bn} = V_p / \underline{-120^{\circ}}$  $\mathbf{V}_{cn} = V_p / \underline{-240^{\circ}} = V_p / \underline{+120^{\circ}}$ 

where  $V_p$  is the effective or rms value of the phase voltages.

This is known as the *abc* sequence or positive sequence. In this phase sequence,  $V_{an}$  leads  $V_{bn}$ , which in turn leads  $V_{cn}$ . This sequence is produced when the rotor (Page8) rotates counterclockwise. As the phasors rotate in the counterclockwise direction with frequency  $\omega$ , they pass through the horizontal axis in a sequence *abcabca*.... Thus, the sequence is *abc* or *bca* or *cab*.



Consider the wye-connected voltages. There are two possible combinations.



This is called the *acb* sequence or negative sequence. In this phase sequence,  $V_{an}$  leads  $V_{cn}$ , which in turn leads  $V_{bn}$ . The *acb* sequence is produced when the rotor (Page8) rotates in the clockwise direction. As the phasors rotate in the counterclockwise direction, they pass the horizontal axis in a sequence acbacba . . . . This describes the *acb* sequence.



 If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltages are said to be balanced. This implies that

 $\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$  $|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$ 

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = V_p / \underline{0^{\circ}} + V_p / \underline{-120^{\circ}} + V_p / \underline{+120^{\circ}}$$
$$= V_p (1.0 - 0.5 - j0.866 - 0.5 + j0.866)$$
$$= 0$$
<sup>12</sup>



A wye- or delta-connected load is said to be unbalanced if the phase impedances are not equal in magnitude or phase.



Two possible three-phase load configurations: (a) a Y-connected load, (b) a  $\Delta$ -connected load.

For a balanced wye-connected load,

 $\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$ 

where  $\mathbf{Z}_{Y}$  is the load impedance per phase. For a *balanced* delta-connected load,

 $\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_{\Delta}$ 

where  $\mathbf{Z}_{\Delta}$  is the load impedance per phase.

$$\mathbf{Z}_{\Delta} = 3\mathbf{Z}_{Y}$$
 or  $\mathbf{Z}_{Y} = \frac{1}{3}\mathbf{Z}_{\Delta}$ 



# 12.2 Balance Three-Phase Voltages(Wye-Delta Transformations)





- Balanced phase voltages are <u>equal in</u> <u>magnitude</u> and are <u>out of phase</u> with each other by 120°.
  - The phase sequence is the <u>time order</u> in which the voltages pass through their respective maximum values.
    - A balanced load is one in which the phase impedances are equal in magnitude and in phase



 $\underline{Ex.1}$  Determine the phase sequence of the set of voltages.

$$v_{an} = 200\cos(\omega t + 10^\circ)$$
$$v_{bn} = 200\cos(\omega t - 230^\circ)$$

$$v_{cn} = 200\cos(\omega t - 110^\circ)$$

#### Solution:

The voltages can be expressed in phasor form as

$$V_{an} = 200∠10^{\circ} V$$
  
 $V_{bn} = 200∠-230^{\circ} V$   
 $V_{cn} = 200∠-110^{\circ} V$ 

We notice that  $V_{an}$  leads  $V_{cn}$  by 120° and  $V_{cn}$  in turn leads  $V_{bn}$  by 120°. Hence, we have an *acb* sequence.



<u>Ex.1.2</u> Given that  $V_{bn} = \frac{110}{30^{\circ}} V$ , find  $V_{an}$  and  $V_{cn}$ , assuming a positive (*abc*) sequence.





Four possible connections

1. Y-Y connection (Y-connected source with a Y-connected load)

2. Y- $\Delta$  connection (Y-connected source with a  $\Delta$ -connected load)

3.  $\Delta$ - $\Delta$  connection

4.  $\Delta$ -Y connection



A balanced Y-Y system is a three-phase system with a balanced y-connected source and a balanced y-connected load.

 $V_L$ =line-to-line voltages ( $V_{ab}$ ,  $V_{bc}$ ,  $V_{ca}$ )







#### <u>Ex.2</u>

Calculate the line currents in the three-wire Y-Y system shown below:



\*Refer to in-class illustration, textbook



#### <u>Solution</u>

We obtain  $I_a$  from the single-phase analysis as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where  $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155/21.8^\circ$ . Hence,

$$\mathbf{I}_a = \frac{110/0^{\circ}}{16.155/21.8^{\circ}} = 6.81/-21.8^{\circ} \text{ A}$$

In as much as the source voltages in Fig. 12.13 are in positive sequence, the line currents are also in positive sequence:

$$\mathbf{I}_{b} = \mathbf{I}_{a} / -120^{\circ} = 6.81 / -141.8^{\circ} \text{ A}$$
$$\mathbf{I}_{c} = \mathbf{I}_{a} / -240^{\circ} = 6.81 / -261.8^{\circ} \text{ A} = 6.81 / 98.2^{\circ} \text{ A}$$

\*Refer to in-class illustration, textbook



A balanced Y- $\Delta$  system is a three-phase system with a balanced y-connected source and a balanced  $\Delta$ -connected load.

Assuming the positive sequence, the phase voltages are

$$\mathbf{V}_{an} = V_p / \underline{0^{\circ}}$$
$$\mathbf{V}_{bn} = V_p / \underline{-120^{\circ}}$$

$$\mathbf{V}_{cn} = V_p / +120^\circ$$

The line voltages are

$$\mathbf{V}_{ab} = \sqrt{3} V_p / 30^\circ = \mathbf{V}_{AB}$$
$$\mathbf{V}_{bc} = \sqrt{3} V_p / -90^\circ = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3} V_p / 150^\circ = \mathbf{V}_{CA}$$

$$I_{L} = \sqrt{3}I_{p} \text{, where}$$
$$I_{L} = |\mathbf{I}_{a}| = |\mathbf{I}_{b}| = |\mathbf{I}_{c}|$$
$$I_{p} = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$





From these voltages, we can obtain the phase currents as

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}}$$

These currents have the same magnitude but are out of phase with each other by 120° by applying KVL around loop *aABbna* gives

$$-\mathbf{V}_{an} + \mathbf{Z}_{\Delta}\mathbf{I}_{AB} + \mathbf{V}_{bn} = 0 \quad \text{or} \quad \mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}$$

The line currents are obtained from the phase currents by applying KCL at nodes *A*, *B*, and *C*. Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Since  $\mathbf{I}_{CA} = \mathbf{I}_{AB} / -240^{\circ}$ ,  $\mathbf{I}_{a} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} (1 - 1 / -240^{\circ}) = \mathbf{I}_{AB} (1 + 0.5 - j0.866) = \mathbf{I}_{AB} \sqrt{3} / -30^{\circ}$ 

showing that the magnitude  $I_L$  of the line current is  $\sqrt{3}$  times the magnitude  $I_p$  of the phase current

$$I_L = \sqrt{3}I_p$$



#### **12.3 Balance Three-Phase Connection** C CA 30° AB 30° 30° la BC

Phasor diagram illustrating the relationship between phase and line currents.

An alternative way of analyzing the Y- $\Delta$  circuit is to transform the  $\Delta$ -connected load to an equivalent Y-connected load. Using the  $\Delta$ -Y transformation formula in Eq. (12.8),





#### <u>Example 3</u>

A balanced abc-sequence Y-connected source with  $(V_{an} = 100 \ge 10^{\circ})$  is connected to a  $\Delta$ -connected load (8+j4)  $\Omega$  per phase. Calculate the phase and line currents.

Solution

Using single-phase analysis,

$$I_a = \frac{V_{an}}{Z_{\Delta}/3} = \frac{100\angle 10^{\circ}}{2.981\angle 26.57^{\circ}} = 33.54\angle -16.57^{\circ} \text{ A}$$

Other line currents are obtained using the abc phase sequence

12.3 Balance Three-Phase Connection A balanced  $\Delta$ - $\Delta$  system is a three-phase system with a balanced  $\Delta$ -connected source and a balanced  $\Delta$ -connected load.





Assuming a positive sequence, the phase voltages for a delta-connected source are

 $\mathbf{V}_{ab} = V_p / \underline{0^{\circ}}$  $\mathbf{V}_{bc} = V_p / \underline{-120^{\circ}}$  $\mathbf{V}_{ca} = V_p / \underline{+120^{\circ}}$ 

The line voltages are the same as the phase voltages. From Fig. 12.17, assuming there is no line impedances, the phase voltages of the delta-connected source are equal to the voltages across the impedances; that is,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Hence, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_{\Delta}} = \frac{\mathbf{V}_{ab}}{Z_{\Delta}}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_{\Delta}} = \frac{\mathbf{V}_{bc}}{Z_{\Delta}}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_{\Delta}} = \frac{\mathbf{V}_{ca}}{Z_{\Delta}}$$



Because the load is delta-connected just as in the previous section, some of the formulas derived there apply here. The line currents are obtained from the phase currents by applying KCL at nodes*A*, *B*, and *C*, as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30°; the magnitude  $I_L$  of the line current is  $\sqrt{3}$  times the magnitude  $I_p$  of the phase current,

$$I_L = \sqrt{3}I_p$$

An alternative way of analyzing the  $\Delta$ - $\Delta$  circuit is to convert both the source and the load to their Y equivalents. We already know that  $\mathbf{Z}_Y = \mathbf{Z}_{\Delta}/3$ . To convert a  $\Delta$ -connected source to a Y-connected source, see the next section.



#### <u>Example 4</u>

A balanced  $\Delta$ -connected load having an impedance 20-j15  $\Omega$  is connected to a  $\Delta$ -connected positive-sequence generator having ( $V_{ab} = 330 \angle 0^{\circ}$  V). Calculate the phase currents of the load and the line currents.

Ans:

The phase currents

 $I_{AB} = 13.2 \angle 36.87^{\circ} \text{ A}; I_{BC} = 13.2 \angle -81.13^{\circ} \text{ A}; I_{AB} = 13.2 \angle 156.87^{\circ} \text{ A}$ The line currents

 $I_a = 22.86 \angle 6.87^{\circ} A; I_b = 22.86 \angle -113.13^{\circ} A; I_c = 22.86 \angle 126.87^{\circ} A$ 

\*Refer to in-class illustration, textbook



### 12.3 Balance Three-Phase Connection A balanced $\Delta$ -Y system is a three-phase system with a balanced y-connected source and a balanced y-connected load.





Consider the  $\Delta$ -Y circuit in Fig. 12.18. Again, assuming the *abc* sequence, the phase voltages of a delta-connected source are

$$\mathbf{V}_{ab} = V_p / \underline{0^{\circ}}, \qquad \mathbf{V}_{bc} = V_p / \underline{-120^{\circ}}$$
$$\mathbf{V}_{ca} = V_p / \underline{+120^{\circ}}$$

These are also the line voltages as well as the phase voltages.

We can obtain the line currents in man y ways. One way is to apply KVL to loop *aANBba* in Fig. 12.18, writing

 $-\mathbf{V}_{ab}+\mathbf{Z}_{Y}\mathbf{I}_{a}-\mathbf{Z}_{Y}\mathbf{I}_{b}=0$ 

or

$$\mathbf{Z}_{Y}(\mathbf{I}_{a}-\mathbf{I}_{b})=\mathbf{V}_{ab}=V_{p}/\underline{0^{\circ}}$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p / 0^\circ}{\mathbf{Z}_Y}$$

But  $\mathbf{I}_b$  lags  $\mathbf{I}_a$  by 120°, since we assumed the *abc* sequence; that is,  $\mathbf{I}_b = \mathbf{I}_a / -120^\circ$ . Hence,



Substituting Eq. (12.36) into Eq. (12.35) gives

$$\mathbf{I}_a = \frac{V_p / \sqrt{3} / -30^\circ}{\mathbf{Z}_Y}$$



Transforming a  $\Delta$ -connected source to an equivalent Y-connected source.



The single-phase equivalent circuit.

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<u>Ex.5</u> A balanced Y-connected load with a phase impedance  $40+j25 \Omega$  is supplied by a balanced, positive-sequence  $\Delta$ -connected source with a line voltage of 210V. Calculate the phase currents. Use  $V_{ab}$  as reference.

Answer

The phase currents

 $I_{AN} = 2.57 \angle -62^{\circ} A;$   $I_{BN} = 2.57 \angle -178^{\circ} A;$  $I_{CN} = 2.57 \angle 58^{\circ} A;$ 



## 12.4 Power in a Balanced System

Comparing the power loss in (a) a single-phase system, and (b) a three-phase system



If <u>same power loss</u> is tolerated in both system, three-phase system use only <u>75%</u> of materials of a single-phase system



## 12.5 Unbalanced Three-Phase Systems

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.



• To calculate power in an unbalanced three-phase system requires that we find the power in each phase.

• The total power is not simply three times the power in one phase but the sum of the powers in the three phases.



## 12.5 Unbalanced Three-Phase Systems

 $\underline{Ex.6}$  Determine the total average power, reactive power, and complex power at the source and at the load



\*Refer to in-class illustration, textbook

## 12.6 Application Residential Wiring



A 120/240 household power system

## 12.6 Application Residential Wiring



#### Single-phase three-wire residential wiring

## 12.6 Application Residential Wiring



A typical wiring diagram of a room