Electrical Engineering 1
12026105

Chapter 10

Sinusoidal Steady-State Analysis

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Analyze electrical circuits in the frequency domain using nodal analysis.
2. Analyze electrical circuits in the frequency domain using mesh analysis.
3. Apply the superposition principle to frequency domain electrical circuits.
4. Apply source transformation in frequency domain circuits.
5. Understand how Thevenin and Norton equivalent circuits can be used in the frequency domain.
6. Analyze electrical circuits with op amps.

Sinusoidal Steady-State Analysis Chapter 10

- Introduction
- Nodal Analysis
- Mesh Analysis
- Superposition Theorem
- Source Transformation
- Thevenin and Norton Equivalent Circuits

10.1 Introduction (1)

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

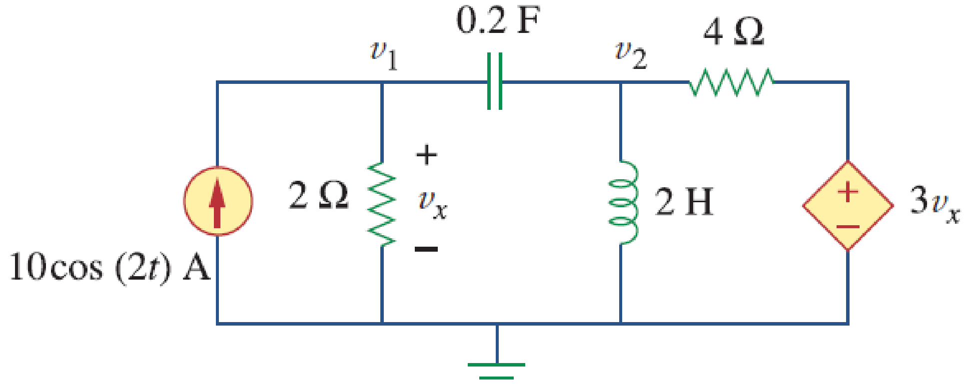
Transform: Time to Freq → Solve: Variables in Freq → Transform: Freq to Time

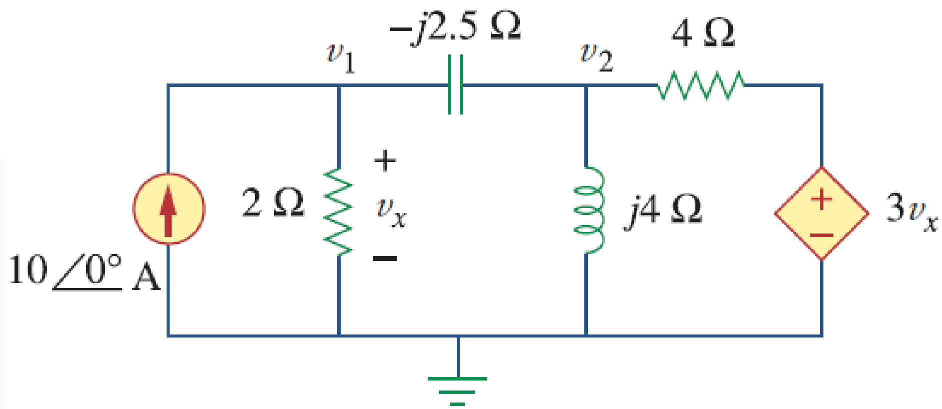
10.1 Introduction (2)

Element	Impedance
R	$Z = R$
L	$Z = j\omega L$
C	$Z = \frac{1}{j\omega C}$

10.2 Nodal Analysis (1)

Ex.1 Using nodal analysis, find v_1 and v_2 in the circuit of figure below.





$$\frac{v_1 - v_2}{-j2.5} + \frac{v_1}{2} = 10\angle 0^\circ \Leftrightarrow (-1 + j1.25)v_1 + v_2 = 25\angle 90^\circ$$

$$\frac{v_2 - v_1}{-j2.5} + \frac{v_2}{j4} + \frac{v_2 - 3v_1}{4} = 0 \Leftrightarrow -4(v_2 - v_1) + 2.5v_2 + j2.5v_2 - j7.5v_1 = 0$$

$$(4 - j7.5)v_1 = (1.5 - j2.5)v_2 \Leftrightarrow v_1 = \left(\frac{24.75 + j1.25}{72.25}\right)v_2$$

$$(-1 + j1.25)\left(\frac{24.75 + j1.25}{72.25}\right)v_2 + v_2 = 25\angle 90^\circ \Leftrightarrow (0.636 + j0.411)v_2 = 25\angle 90^\circ$$

$$(0.757\angle 32.8^\circ)v_2 = 25\angle 90^\circ$$

$$v_2 = \frac{25\angle 90^\circ}{0.757\angle 32.8^\circ} = 33.02\angle 57.2^\circ$$

$$v_1 = \left(\frac{24.75 + j1.25}{72.25}\right)33.02\angle 57.2^\circ$$

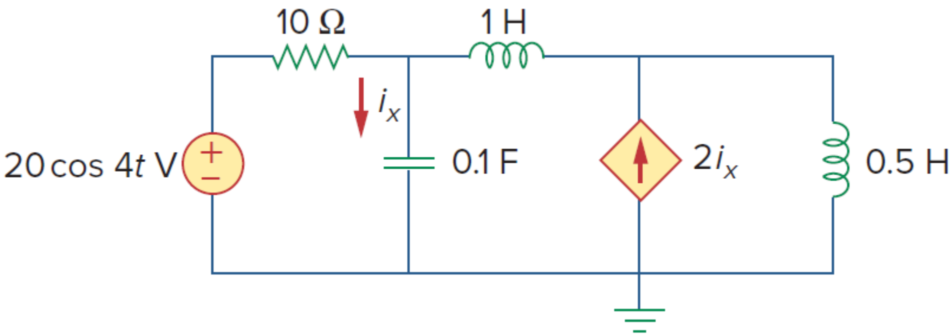
$$= (0.343\angle 2.89^\circ)(33.02\angle 57.2^\circ)$$

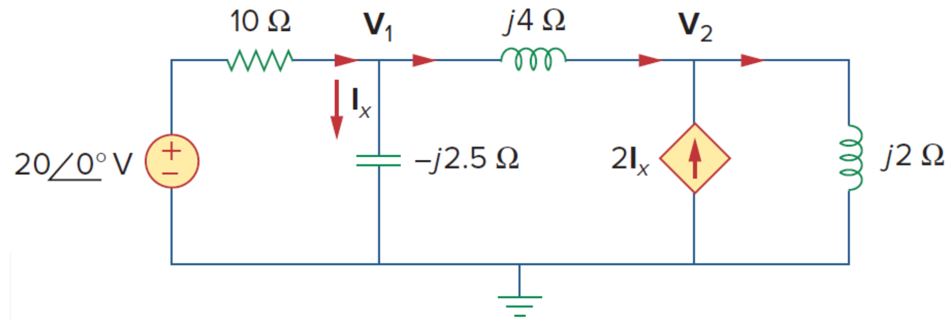
$$= 11.32\angle 60.09^\circ$$

Answer: $v_1(t)$ = **11.32 cos(2t + 60.01°) V** $v_2(t)$ = **33.02 cos(2t + 57.12°) V**

10.2 Nodal Analysis (2)

Ex.2 Find i_x in the circuit of Fig. using nodal analysis.





Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

Equations can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\,\underline{/18.43^\circ}\text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\,\underline{/198.3^\circ}\text{ V}$$

The current $\mathbf{I}_x$ is given by

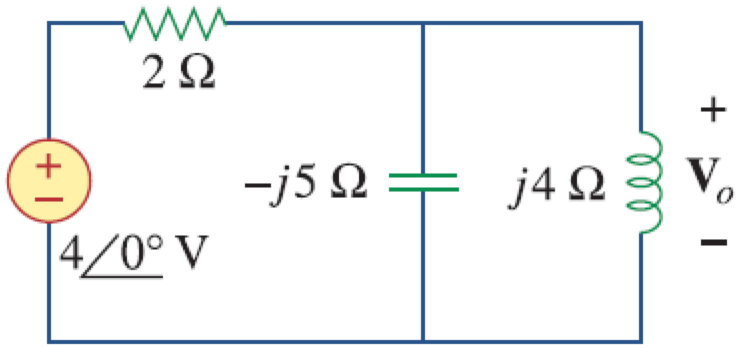
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\,\underline{/18.43^\circ}}{2.5\,\underline{/-90^\circ}} = 7.59\,\underline{/108.4^\circ}\text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ)\text{ A}$$

10.3 Mesh Analysis (1)

Ex.3 Find V_0 using node analysis.



$$\frac{V_0}{j4}+\frac{V_0}{-j5}+\frac{V_0-4}{2}=0$$

$$V_0\left(\frac{1}{j4}+\frac{1}{-j5}+\frac{1}{2}\right)=2$$

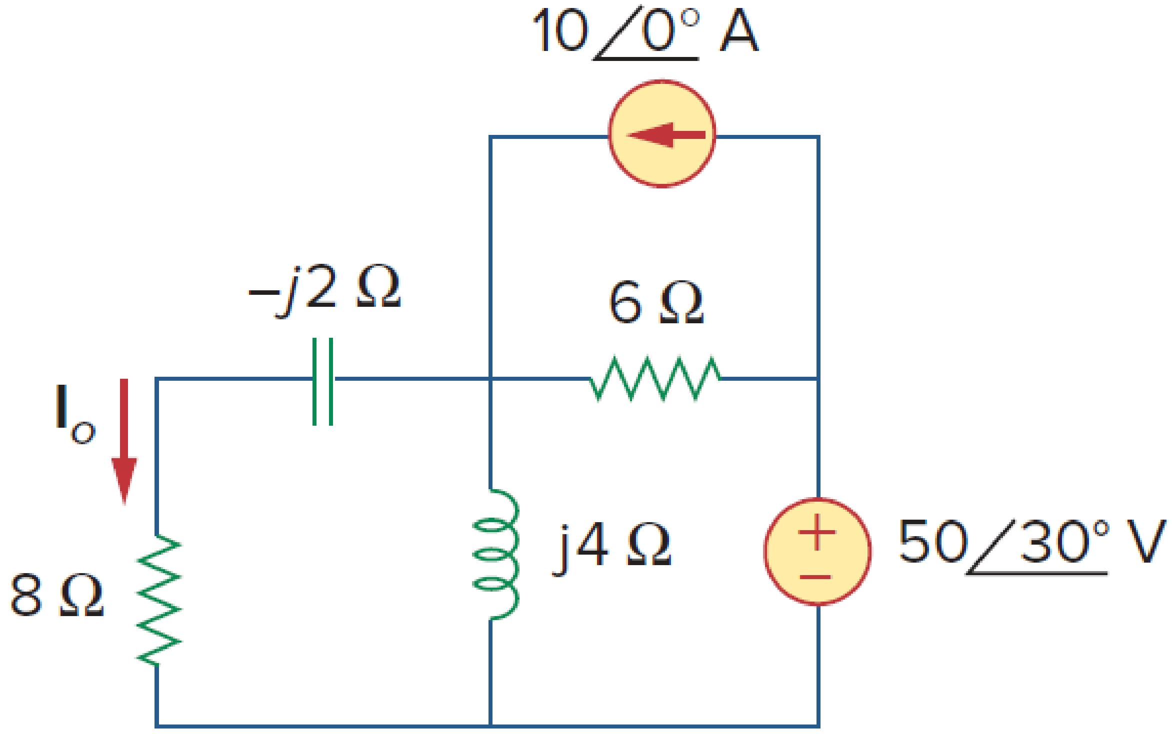
$$V_0\left(-j0.25+j0.2+0.5\right)=2$$

$$V_0=\frac{2}{0.5-j0.05}=\frac{2}{0.5-j0.05}\times\frac{0.5+j0.05}{0.5+j0.05}=\frac{1+j0.1}{0.2525}=3.98\angle 5.71^{\circ}\,A$$

Answer: $V_0=3.98\angle 5.71^{\circ}\,A$

10.3 Mesh Analysis (2)

Ex.4 Find I_o using mesh analysis.



Answer: $5.969\angle 65.45^\circ$ A.

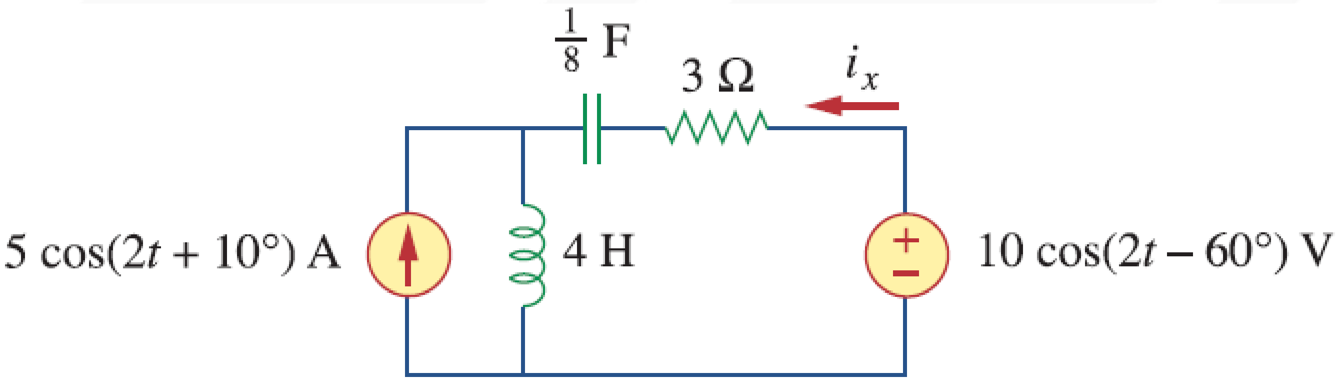
10.4 Superposition Theorem (1)

The theorem becomes important if the circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.4 Superposition Theorem (2)

Ex.5 Calculate i_x in the circuit of figure shown below using the superposition theorem.



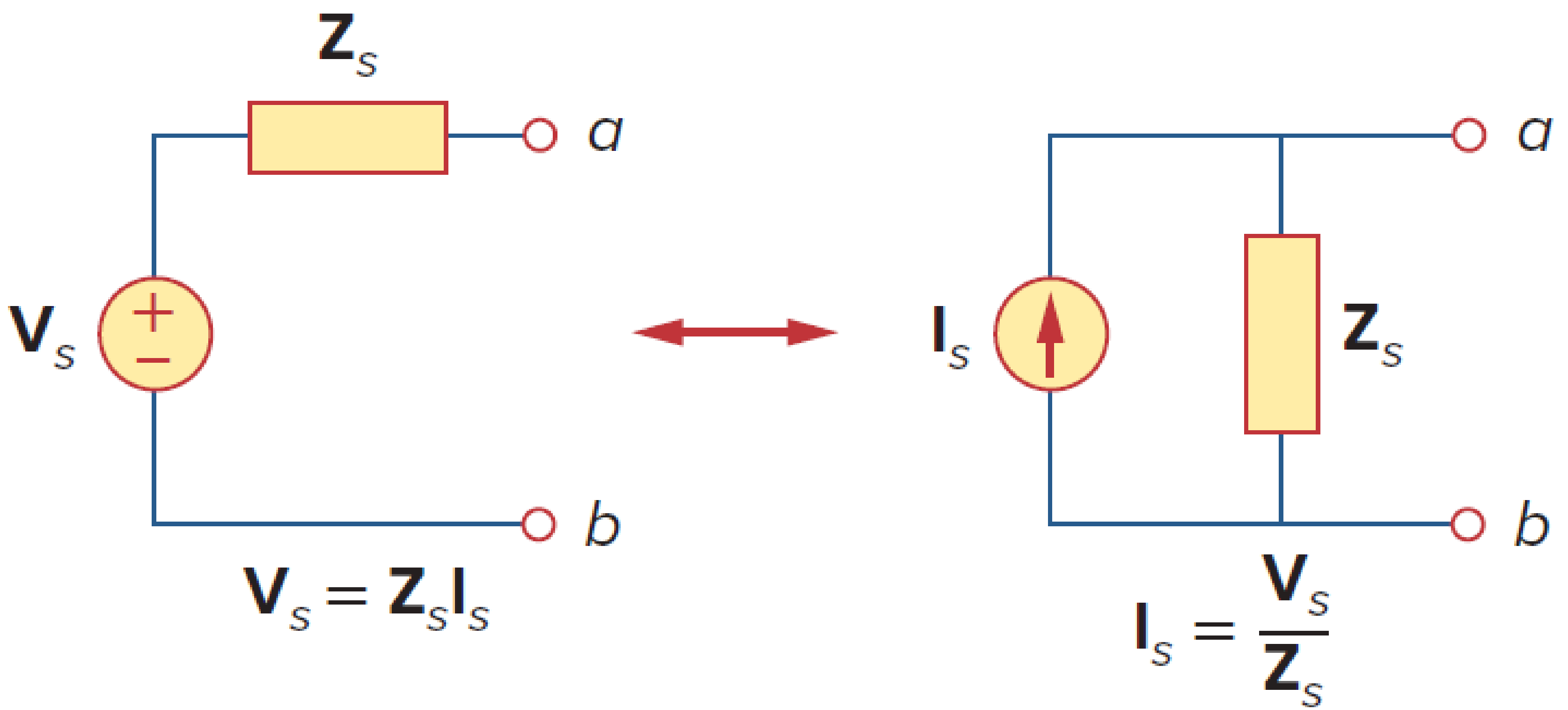
$$(i_x)_1 = \frac{10\cos(2t-60°)}{3+j8-j4} = \frac{(10\angle-60°)(5\angle-53.13°)}{25} = 2\angle-113.13° = -0.7856 - j1.839$$

$$(i_x)_2 = -\frac{j8}{3+j8-j4}5\angle10° = \frac{-32-j24}{25}5\angle10° = 1.6\angle216.87° \times 5\angle10° = 8\angle226.87° = -5.47 - j5.84$$

$$i_x = (i_x)_1 + (i_x)_2 = -6.26 - j7.68 = 9.9\angle-129°$$

Answer : $i_x = 9.902\cos(2t - 129.17°)A$

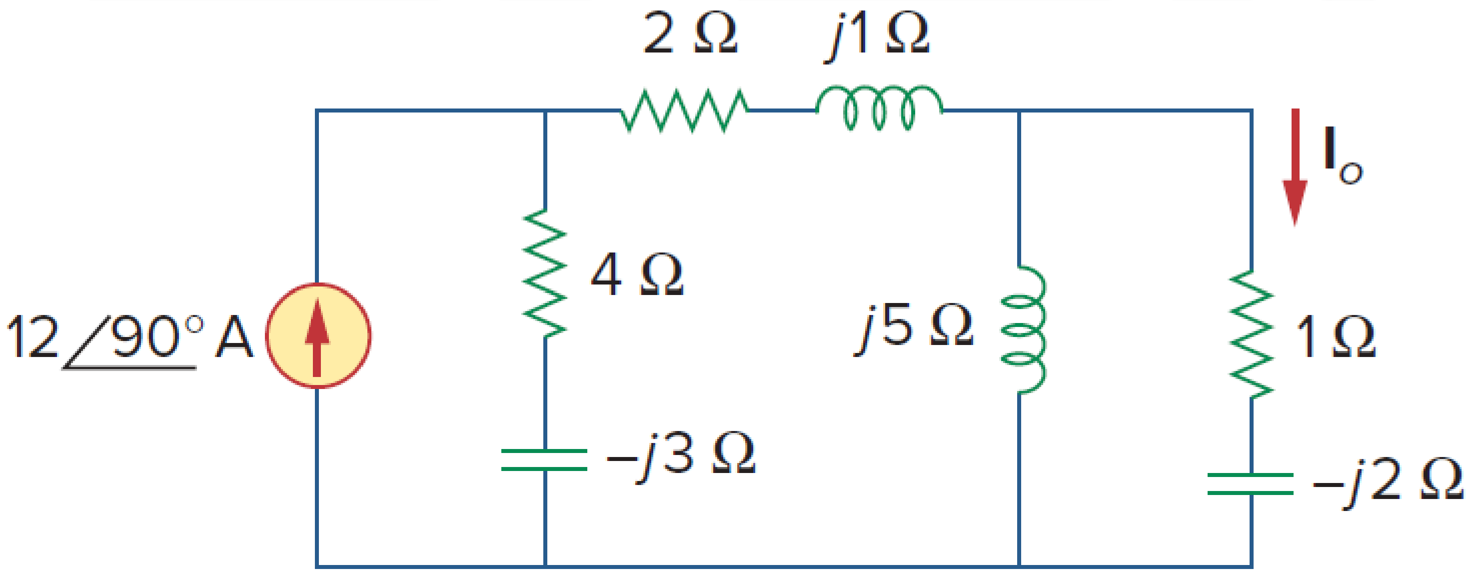
10.5 Source Transformation (1)



Source transformation.

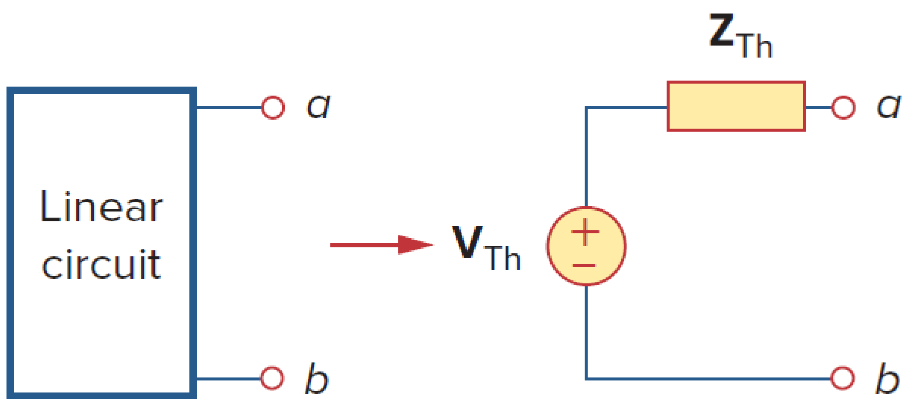
10.5 Source Transformation (2)

Ex.6 Find I_o in the circuit of figure below using the concept of source transformation.

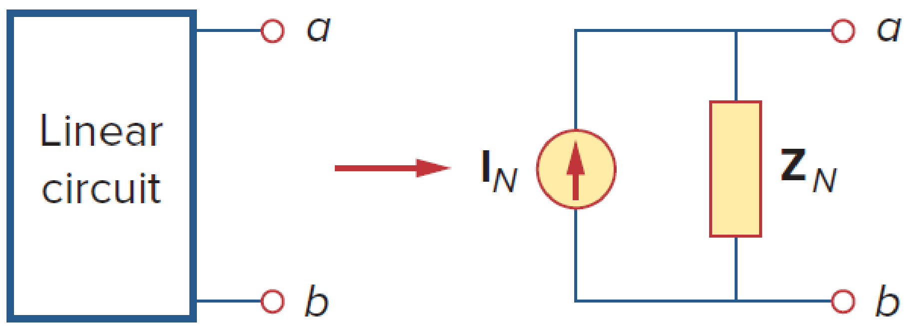


Answer: $9.863\angle 99.46^\circ$ A.

10.6 Thevenin and Norton Equivalent Circuits (1)

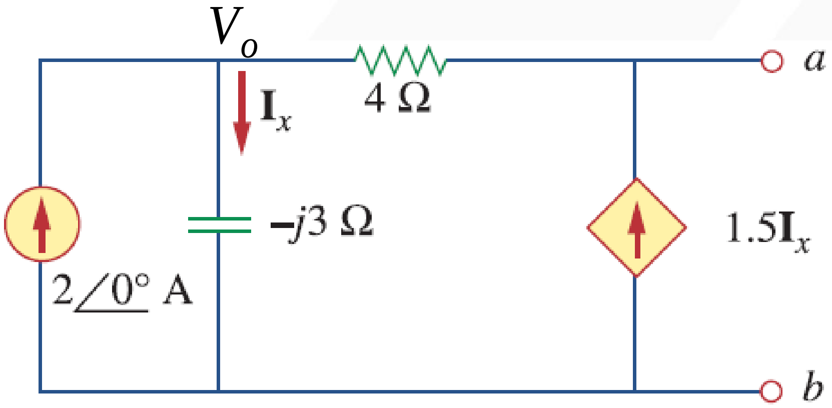


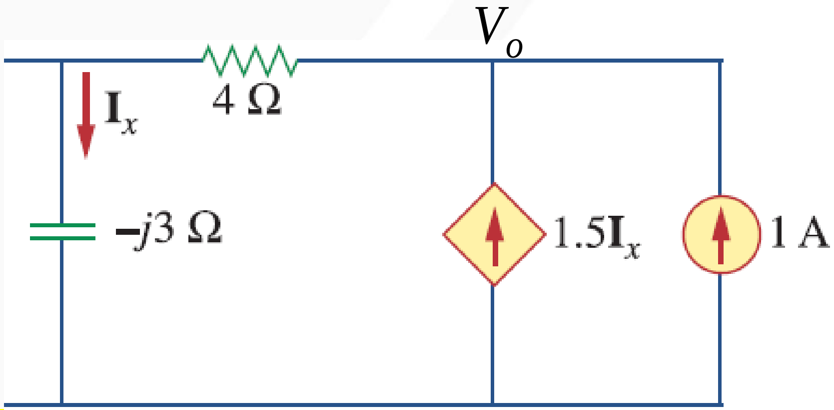
Thevenin equivalent.



Norton equivalent.

Ex.7 Find the Thevenin equivalent at terminals a–b of the circuit below.





$$2+1.5I_x = I_x \rightarrow I_x = -4$$

$$V_o = I_x(-j3) = j12$$

$$V_{TH} = V_o + 6I_x = -24 + j12$$

$$1+1.5I_x = I_x \rightarrow I_x = -2$$

$$V_o = I_x(4-j3) = -8 + j6$$

$$Z_{TH} = \frac{V_o}{1} = -8 + j6$$

$$Z_{TH} = -8 + j6\Omega$$

$$V_{TH} = (-24 + j12)V$$