Sensors and Actuators
(เซนเซอร์และตัวขับเร้า)

Chapter 3: ACTUATORS&TRANSDUCERS

By

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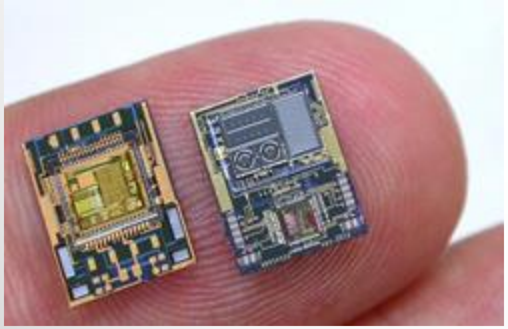


ACTUATORS & TRANSDUCERS

- The energy conversion schemes presented here include electrostatic, piezoelectric, electrostrictive, magnetostrictive, electromagnetic, electrodynamic, and electrothermal. Most of the schemes are reciprocal and hence these devices are generally referred to as transducers.

- Although some of these schemes are not quite amenable for smart micromechanical systems, they do have the potential for being used in such systems in the foreseeable future.

- One important step in the design of these mechanical systems is obtaining their electrical equivalent circuits from analytical models.

- These equivalent circuits are neither unique nor exact, but would serve as an easily understood tool in transducer design.

ELECTROSTATIC TRANSDUCERS

- Electrostatic actuation is the most common type of electromechanical energy conversion scheme in micromechanical systems.

- The structure of this type of transducer commonly consists of a capacitor arrangement, where one of the plates is movable by the application of a bias voltage. This produces displacement, a mechanical form of energy.

- The electromechanical force in a simple (fixed) parallel plate capacitor: $F = \frac{1}{2} v^2 \frac{\varepsilon A}{x^2}$ ①

- In more complicated systems, it is difficult to calculate this directly. Instead, we start with the basic energy balance equation: $\mathrm{d}W_{\mathrm{es}} + \mathrm{d}W_{\mathrm{m}} = \mathrm{d}W_{\mathrm{e}}$ ②

- This expression indicates that the force balance is between the electrostatic and mechanical forces.

$$VI\mathrm{d}t + F\mathrm{d}x = \mathrm{d}\left(\frac{1}{2}CV^2\right)$$ ③

V=Kq/r

Energy=Kq_1q_2/r

energy stored in capacitor

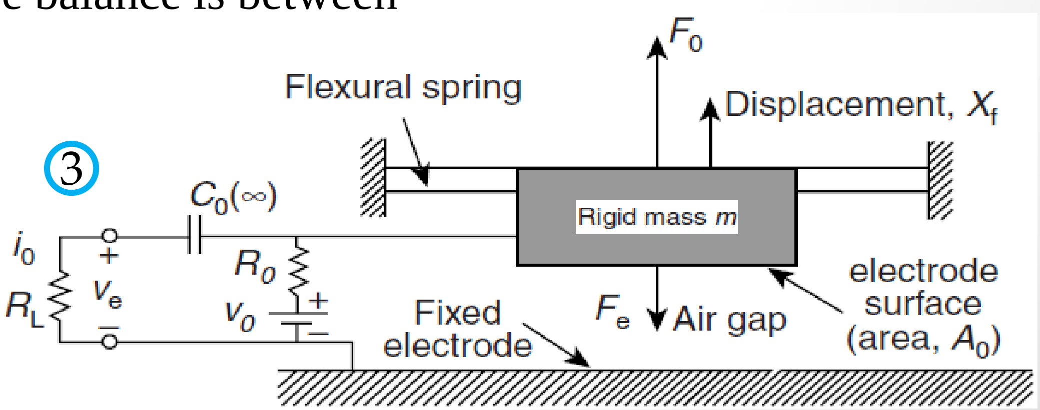


Figure 1. Schematic of a practical electrostatic transducer.

ELECTROSTATIC TRANSDUCERS

➢The capacitance of the arrangement cannot be considered a constant. Eliminating I :

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt} \quad (4)$$

➢The first term is for a fixed capacitor, while the second term results from the physical motion of the movable plate. Obviously, this is zero for a fixed plate capacitor. Equation (3) becomes :

$$VCdV + V^2dC + Fdx = CVdV + \frac{1}{2}V^2dC \quad (5)$$

$$\boxed{C = \frac{\varepsilon A}{x} = \varepsilon A x^{-1} \qquad \frac{dC}{dx} = \frac{d(\varepsilon A x^{-1})}{dx} = -\frac{\varepsilon A}{x^2}}$$

$$Fdx = -\frac{1}{2}V^2dC \quad (6)$$

$$F = -\frac{1}{2}V^2\frac{dC}{dx} \quad (7)$$

➢dC/dx is negative for a parallel plate capacitor. Furthermore, the force depends on the square of the voltage and hence does not depend on its polarity.

➢When both plates of the capacitor are fixed, there is no mechanical motion, and hence no work is done: $VCdV + 0 + 0 = CVdV + 0$ (8), so the term CVdV represents the energy stored!!

$$VIdt + Fdx = d\left(\frac{1}{2}CV^2\right)$$

ELECTROSTATIC TRANSDUCERS

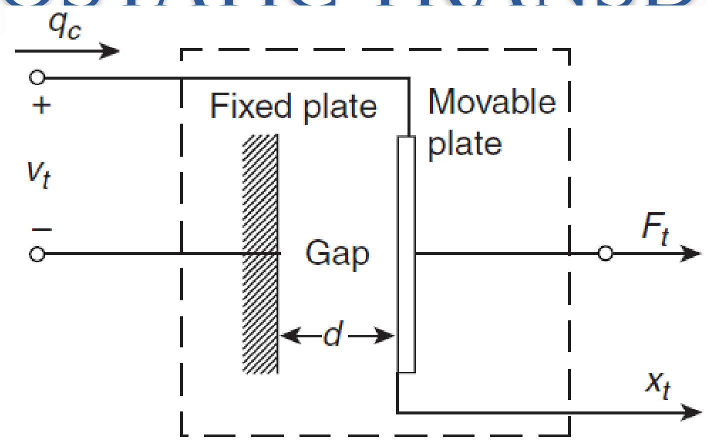


Figure 2. Schematic of a simplified case for an electrostatic transducer.

- By cancelling term energy stored, obtaining the energy transfer caused entirely by motion:

$$V^2 \mathrm{d}C + F\mathrm{d}x = \frac{1}{2} V^2 \mathrm{d}C \quad (9)$$

- The electrical source contributes twice as much energy as the mechanical source.

- Based on the simplified schematic of the transducer shown in Figure 2, constitutive equations can be derived; the state variables of this are the displacement x_t and charge q_t . Since all variables are dependent on time, these are omitted here for convenience. The electrical energy contained in the transducer is given by the following:

$$W_e = W_e(q_t, x_t) = \frac{q_t^2}{2\underbrace{C(x_t)}_{\text{fn of } x_t}} = \frac{q_t^2(d + x_t)}{2\varepsilon_0 A_e} \quad (10)$$

d, the spacing of the plates when uncharged.

ELECTROSTATIC TRANSDUCERS

➢The total differential of W_e is:

$$\mathrm{d}(uv) = u\mathrm{d}v + v\mathrm{d}u$$

$$\mathrm{d}W_\mathrm{e} = \left(\frac{\partial W_\mathrm{e}}{\partial q_t}\right)_{x_t = \text{constant}} \mathrm{d}q_t + \left(\frac{\partial W_\mathrm{e}}{\partial x_t}\right)_{q_t = \text{constant}} \mathrm{d}x_t \qquad 11$$

➢In equilibrium, the energy put into the transducer through the electric and mechanical ports is given by:

$$\mathrm{d}W_\mathrm{e} = v_t\,\mathrm{d}q_t + F_t\,\mathrm{d}x_t \qquad 12$$

$$v_t\,(q_t, x_t) \equiv \left.\frac{\partial W_\mathrm{e}(q_t, x_t)}{\partial q_t}\right|_{x_t = \text{constant}} = \frac{q_t(d + x_t)}{\varepsilon_0 A_\mathrm{e}} \qquad 13$$

$$F_t(q_t, x_t) \equiv \left.\frac{\partial W_\mathrm{e}(q_t, x_t)}{\partial x_t}\right|_{q_t = \text{constant}} = \frac{q_t^2}{2\varepsilon_0 A_\mathrm{e}} \qquad 14$$

➢The above equations define the terminal voltage and the force as being the effort variables at the respective ports. The equilibrium values are given by the partial derivatives of W_e with respect to the corresponding state variable (the displacement x_t and charge q_t). Note that F_t is the externally applied force necessary to achieve equilibrium. Its magnitude is equal to the electrostatic Coulomb force between plates of a charged capacitor (opposite in direction).

$$W_\mathrm{e} = W_\mathrm{e}(q_t, x_t) = \frac{q_t^2}{2C(x_t)} = \frac{q_t^2(d + x_t)}{2\varepsilon_0 A_\mathrm{e}}$$

ELECTROSTATIC TRANSDUCERS

➢This force has a quadratic dependence with charge. To make it linear, we assume small signal state variables. So:

$$x_t = x_0 + x(t) \quad (15)$$

$$q_t = q_0 + q(t) \quad (16)$$

$$v(q,x) = \left.\frac{\partial v_t}{\partial q_t}\right|_0 q + \left.\frac{\partial v_t}{\partial x_t}\right|_0 x = \frac{(d+x_0)}{\varepsilon_0 A_e} q + \frac{q_0}{\varepsilon_0 A_e} x = \frac{1}{C_0} q + \frac{v_0}{x_0} x \quad (17)$$

$$F(q,x) = \left.\frac{\partial F_t}{\partial q_t}\right|_0 q + \left.\frac{\partial F_t}{\partial x_t}\right|_0 x = \frac{q_0}{\varepsilon_0 A_e} q + 0x = \frac{v_0}{x_0} q + 0x \quad (18)$$

t=0

➢Note that bias signals are independent of time since they define static equilibrium. It is rather easy to show that the plate illustrated in Figure 3(a) is not in equilibrium. To keep the plate in place we need to provide an external force. This requires a spring constant term, corresponding to the mechanical energy at the spring, added to Equation (10):

$$v_t(q_t, x_t) \equiv \left.\frac{\partial W_e(q_t, x_t)}{\partial q_t}\right|_{x_t = \text{constant}} = \frac{q_t(d+x_t)}{\varepsilon_0 A_e}$$

$$F_t(q_t, x_t) \equiv \left.\frac{\partial W_e(q_t, x_t)}{\partial x_t}\right|_{q_t = \text{constant}} = \frac{q_t^2}{2\varepsilon_0 A_e}$$

ELECTROSTATIC TRANSDUCERS

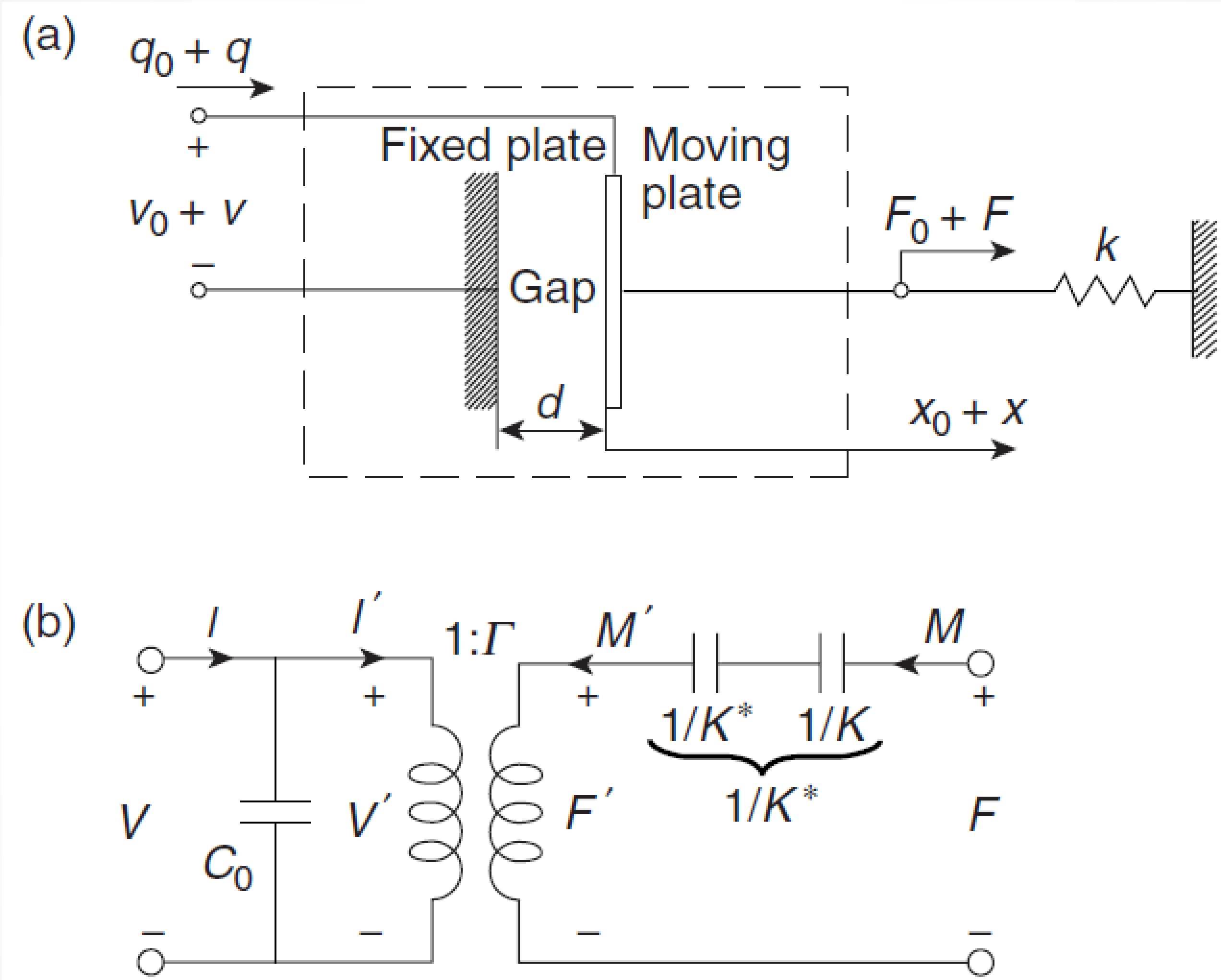


Figure 3. Schematic (a) and equivalent circuit (b) of an electrostatic actuator with a spring attached to the movable plate for stability.

ELECTROSTATIC TRANSDUCERS

➢This force has a quadratic dependence with charge. To make it linear, we assume small signal state variables. So:

$$W_{\mathrm{em}} = W_{\mathrm{em}}(q_t, x_t) = \frac{q_t^2}{2c(x_t)} + \frac{1}{2}k(x_t - x_r)^2 = \frac{q_t^2(d + x_t)}{2\varepsilon_0 A_{\mathrm{e}}} + \frac{1}{2}k(x_t - x_r)^2 \quad 19$$

➢This changes Equations 14 and 18 to the following:

$$F_t(q_t, x_t) \equiv \left.\frac{\partial W_{\mathrm{em}}(q_t, x_t)}{\partial x_t}\right|_{q_t = \mathrm{constant}} = \frac{q_t^2}{2\varepsilon_0 A_{\mathrm{e}}} + kx_t \quad 20$$

$$F(q, x) = \left.\frac{\partial F_t}{\partial qt}\right|_0 q + \left.\frac{\partial F_t}{\partial xt}\right|_0 x = \frac{q_0}{\varepsilon_0 A_{\mathrm{e}}} q + kx = \frac{v_0}{x_0} q + kx \quad 21$$

➢Note that Equations 17 and 21 express voltage and force in terms of displacement and charge. It is usually required to have voltage and displacement as the independent variables. This makes:

$$q(v, x) = \frac{\varepsilon_0 A_{\mathrm{e}}}{d + x_0} v - \frac{q_0}{d + x_0} x = \frac{\varepsilon_0 A_{\mathrm{e}}}{d + x_0} v - \frac{\varepsilon_0 A_{\mathrm{e}} v_0}{(d + x_0)^2} x \quad 22$$

$$F(v, x) = \frac{q_0}{d + x_0} v + \left(k - \frac{q_0^2}{\varepsilon_0 A_{\mathrm{e}}(d + x_0)}\right) x = \frac{\varepsilon_0 A_{\mathrm{e}} v_0}{(d + x_0)^2} v + \left(k - \frac{\varepsilon_0 A_{\mathrm{e}} v_0^2}{(d + x_0)^3}\right) x \quad 23$$

ELECTROSTATIC TRANSDUCERS

➢The system is in equilibrium as long as the second term on the right-hand side of Equation (23) is negative.

$$k < k', \quad \text{where} \quad k' = \frac{\varepsilon_0 A_e v_0^2}{(d + x_0)^3}$$

➢The matrix form of Equations (17) and (21) is:

$$\begin{bmatrix} v \\ F \end{bmatrix} = \begin{bmatrix} \dfrac{d + x_0}{\varepsilon_0 A_e} & \dfrac{q_0}{\varepsilon_0 A_e} \\ \dfrac{q_0}{\varepsilon_0 A_e} & k \end{bmatrix} \begin{bmatrix} q \\ x \end{bmatrix} \qquad (24)$$

➢The static capacitance and transduction factor are:

$$C_0 = \frac{\varepsilon_0 A_e}{d + x_0}; \; \Gamma = \frac{q_0}{d + x_0}$$

➢Therefore, Equation (24) becomes :

$$\begin{bmatrix} v \\ F \end{bmatrix} = \begin{bmatrix} \dfrac{1}{C_0} & \dfrac{\Gamma}{C_0} \\ \dfrac{\Gamma}{C_0} & k \end{bmatrix} \begin{bmatrix} q \\ x \end{bmatrix} \qquad (25)$$

ELECTROSTATIC TRANSDUCERS

➢The 2×2 matrix in Equation (25) is the constitutive matrix for the electrostatic transducer. The coupling factor K is an important characteristic of an electromechanical transducer. This gives the electromechanical energy conversion for a lossless transducer:

$$K = \sqrt{\frac{\Gamma^2}{kC_0}} \qquad 26$$

➢A stable equilibrium state exists for 0 < K < 1. Typical values for 0.05 < K < 0.25.

➢Transduction may also be expressed in such a way as to connect between electrical variables (on the left-hand side) and mechanical variables (on the right-hand side). The transfer matrix relates force (F) and velocity (x') with voltage (v) and current (I).

➢We start with rewriting the second part of Equation (25) with *q* on the left-hand side and taking the time derivative for current:

$$I = j\omega \frac{C_0}{\Gamma} F - \frac{kC_0}{\Gamma} U \qquad 27$$

➢We assume time-harmonic variations in the force and substitute velocity for the time derivative of displacement. Substituting this into the first part of Equation (25):

ELECTROSTATIC TRANSDUCERS

$$v = \frac{1}{C_0}\left(\frac{C_0}{\Gamma}F - \frac{kC_0}{\Gamma}x\right) + \frac{\Gamma}{C_0}x \quad = \frac{1}{\Gamma}F + \left(\frac{\Gamma^2}{C_0} - k\right)\frac{U}{j\omega\Gamma} \qquad (28)$$

$$\begin{bmatrix} v \\ I \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\Gamma} & \dfrac{1}{j\omega\Gamma}\left(\dfrac{\Gamma^2}{C_0} - k\right) \\ j\omega\dfrac{C_0}{\Gamma} & -\dfrac{kC_0}{\Gamma} \end{bmatrix} \begin{bmatrix} F \\ U \end{bmatrix} \qquad (29)$$

➢This 2×2 matrix is known as the transfer matrix. This transfer can be split as follows to conveniently express the equivalent circuit for the transducer :

$$\begin{bmatrix} \dfrac{1}{\Gamma} & \dfrac{1}{j\omega\Gamma}\left(\dfrac{\Gamma^2}{C_0} - k\right) \\ j\omega\dfrac{C_0}{\Gamma} & -\dfrac{kC_0}{\Gamma} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C_0 & 1 \end{bmatrix} \times \begin{bmatrix} \dfrac{1}{\Gamma} & 0 \\ 0 & -\Gamma \end{bmatrix} \begin{bmatrix} 1 & \dfrac{1}{j\omega}\left(\dfrac{\Gamma^2}{C_0} - k\right) \\ 0 & 1 \end{bmatrix} \qquad (30)$$

ELECTROSTATIC TRANSDUCERS

➢As noted earlier, the spring is represented in the circuit by a capacitor. The corresponding "impedance" of the spring (= force/velocity) is k/jω. The spring has a negative stiffness, as follows:

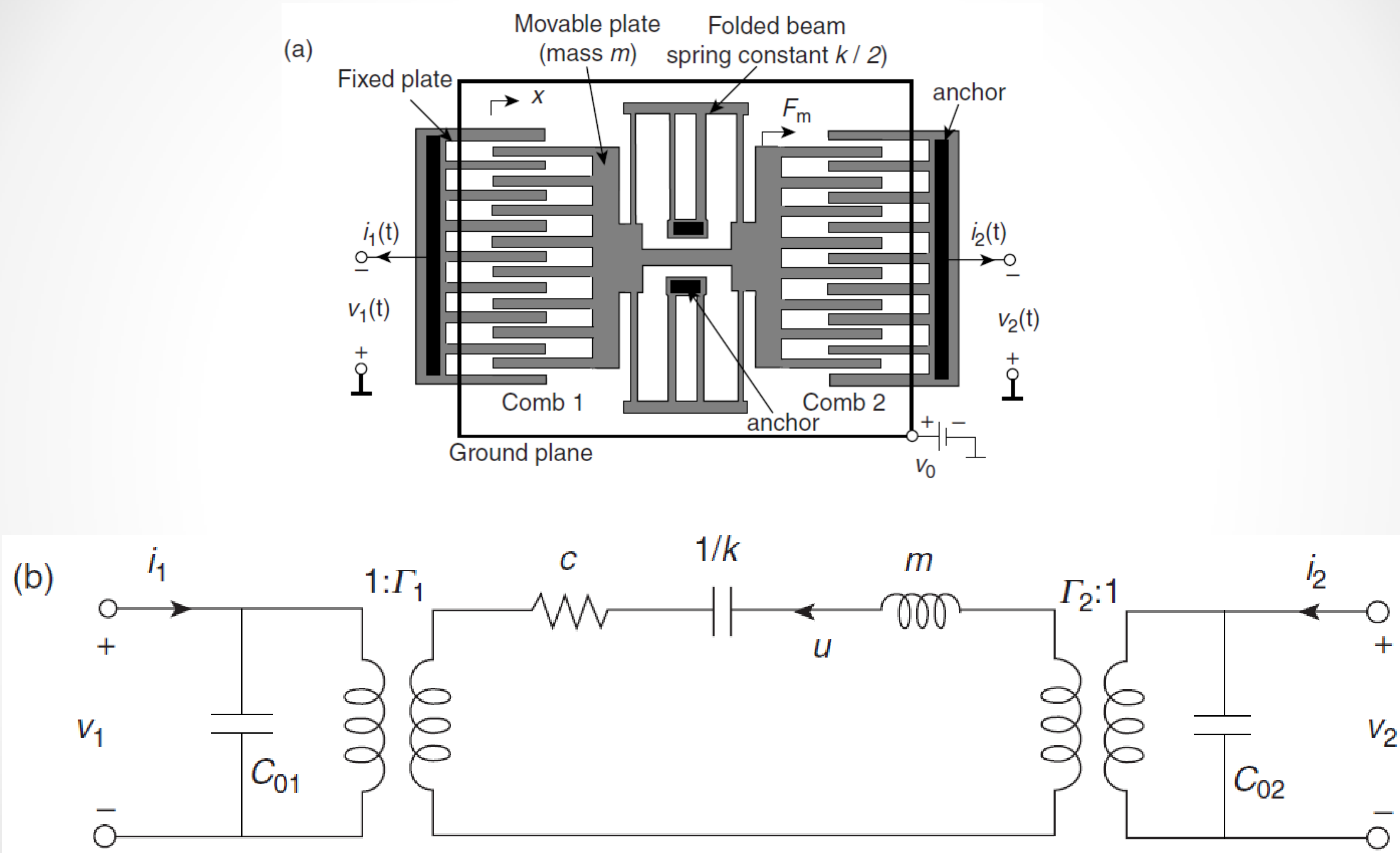
$$-k' = -\frac{\Gamma^2}{C_0} = -\frac{\varepsilon_0 A_e v_0^2}{(d + x_0)^3} = -K^2 k \qquad 31$$

➢This is a result of the electromechanical coupling, leading to a lowering of the overall dynamic spring constant.

➢If we combine the two springs, the combined spring constant is: $k^* = k(1 - K^2)$ 32

➢Recall that the system is mechanically stable as long as this spring constant is positive, i.e. $K < 1$. If the coupling K is zero (K=0), $k^*=k$. Therefore, k is the measured stiffness when the electrical port is short-circuited and k is the stiffness when it is open-circuited.

ELECTROSTATIC TRANSDUCERS

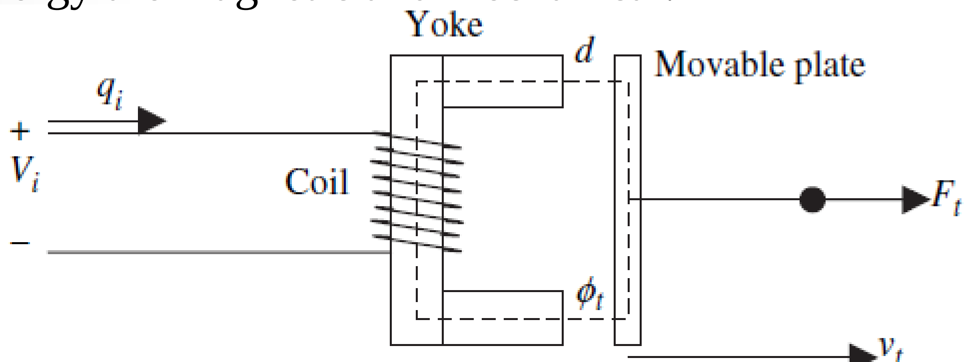


Schematic (a) and equivalent circuit (b) of a comb-type electrostatic resonator.

ELECTROMAGNETIC TRANSDUCERS

➢The magnetic counterpart of a moving plate capacitor is a moving coil inductor (energy-storing transducer).

➢The forms of energy are magnetic and mechanical.



➢When a current i flows through the coil, the magnetic flux is ϕ_t .

➢The magnetic flux in the core (ϕ_t) is related to the current through the coil by:

$$\phi_t = L(x_t)i_t \qquad (1)$$

➢The magnetic energy stored in the transducer when an input is applied to it is given by:

$$W_{\mathrm{M}} = \frac{1}{2}L(x_t)\,i^2 \qquad (2)$$

➢where $L(x_t)$ is the inductance of the driving coil when the moving coil is at $x=x_t$. Therefore:

$$L(x_t) = \frac{N\mu A_{\mathrm{e}}}{d + x_t} \qquad (3)$$

➢N is the number of turns, μ is the permeability and A_e is the effective area of the movable plate.

ELECTROMAGNETIC TRANSDUCERS

$$W_{\mathrm{M}} = \frac{1}{2}\frac{\phi_t^2}{L(x_t)} = \frac{\phi_t^2(d + x_t)}{2N^2\mu A_{\mathrm{e}}} \qquad 4$$

- This shows that W_{M} is a function of ϕ_t and x_t . Therefore we can write:

$$\mathrm{d}W_{\mathrm{M}} = \left.\frac{\partial W_{\mathrm{M}}}{\partial \phi_t}\right|_{x_t = \mathrm{constant}} \mathrm{d}\phi_t + \left.\frac{\partial W_{\mathrm{M}}}{\partial x_t}\right|_{\phi_t = \mathrm{constant}} \qquad 5$$

$$\mathrm{d}x_t = i_t \mathrm{d}\phi_t + F_t \mathrm{d}x_t$$

- From this, we can get

$$i_t(\phi_t, x_t) = \frac{\partial W_{\mathrm{M}}(\phi_t, x_t)}{\partial \phi_t} = \frac{\phi_t(d + x_t)}{N^2\mu A_{\mathrm{e}}} \qquad 6$$

$$F_t(\phi_t, x_t) = \frac{\partial W_{\mathrm{M}}(\phi_t, x_t)}{\partial x_t} = \frac{\phi_t^2}{N^2\mu A_{\mathrm{e}}} \qquad 7$$

ELECTROMAGNETIC TRANSDUCERS

- Note that the force-to-flux relationship here is quadratic. To linearize this, we assume small signal conditions:

$$x_t = x_0 + x(t) \quad (8)$$

$$\phi_t = \phi_0 + \phi(t) \quad (9)$$

- Therefore,

$$i(\phi, x) = \left.\frac{\partial i_t}{\partial \phi_t}\right|_{x=0} \phi + \left.\frac{\partial i_t}{\partial x_t}\right|_{\phi=0} x = \frac{d + x_0}{N^2 \mu A_e} \phi + \frac{\phi_0}{N^2 \mu A_e} x \quad (10)$$

$$F(\phi, x) = \left.\frac{\partial F_t}{\partial \phi_t}\right|_{x=0} \phi + \left.\frac{\partial F_t}{\partial x_t}\right|_{\phi=0} x = \frac{\phi_0}{N^2 \mu A_e} \phi + 0x \quad (11)$$

- These are the constitutive relationships for the transducer. As discussed in the case of the electrostatic transducer, an additional element is required to keep the plate in a stable equilibrium. The spring element for this purpose is attached to the movable plate, as shown in Figure 1. The modified energy-balance equation is:

$$W'_{\mathrm{M}} = W'_{\mathrm{M}}(\phi_t, x_t) = \frac{\phi_t^2}{2L(x_t)} + \frac{1}{2} k (x_t - x_r)^2 \quad (12)$$